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Monopole Impedance and Gain Measurements of Finite Ground Planes

NIJ Report 200–87

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Monopole Impedance and Gain Measurements of Finite Ground Planes

NIJ Report 200-87

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FOREWORD

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Technical comments and suggestions concerning this report are invited from all interested parties. They may be addressed to the Law Enforcement Standards Laboratory, National Bureau of Standards, Gaithersburg, MD 20899.

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COMMONLY USED SYMBOLS AND ABBREVIATIONS

A	ampere	H	henry	nm	nanometer
ac	alternating current	h	hour	No.	number
AM	amplitude modulation	hf	high frequency	o.d.	outside diameter
cd	candela	Hz	hertz (c/s)	Ω	ohm
cm	centimeter	i.d.	inside diameter	p.	page
CP	chemically pure	in	inch	Pa	pascal
c/s	cycle per second	ir	infrared	pe	probable error
d	day	J	joule	pp.	pages
dB	decibel	L	lambert	ppm	part per million
dc	direct current	L	liter	qt	quart
°C	degree Celsius	lb	pound	rad	radian
°F	degree Fahrenheit	lbf	pound-force	rf	radio frequency
diam	diameter	lbf·in	pound-force inch	rh	relative humidity
emf	electromotive force	lm	lumen	s	second
eq	equation	ln	logarithm (natural)	SD	standard deviation
F	farad	log	logarithm (common)	sec.	section
fc	footcandle	*M*	molar	SWR	standing wave radio
fig.	figure	m	meter	uhf	ultrahigh frequency
FM	frequency modulation	min	minute	uv	ultraviolet
ft	foot	mm	millimeter	V	volt
ft/s	foot per second	mph	mile per hour	vhf	very high frequency
g	acceleration	m/s	meter per second	W	watt
g	gram	N	newton	λ	wavelength
gr	grain	N·m	newton meter	wt	weight

area = unit^2 (e.g., ft^2, in^2, etc.); volume = unit^3 (e.g., ft^3, m^3, etc.)

PREFIXES

d	deci (10^{-1})	da	deka (10)
c	centi (10^{-2})	h	hecto (10^2)
m	milli (10^{-3})	k	kilo (10^3)
μ	micro (10^{-6})	M	mega (10^6)
n	nano (10^{-9})	G	giga (10^9)
p	pico (10^{-12})	T	tera (10^{12})

COMMON CONVERSIONS

(See ASTM E380)

ft/s × 0.3048000 = m/s
ft × 0.3048 = m
ft·lbf × 1.355818 = J
gr × 0.06479891 = g
in × 2.54 = cm
kWh × 3 600 000 = J
lb × 0.4535924 = kg
lbf × 4.448222 = N
lbf/ft × 14.59390 = N/m
lbf·in × 0.1129848 = N·m
lbf/in^2 × 6894.757 = Pa
mph × 1.609344 = km/h
qt × 0.9463529 = L

Temperature: $(T_{°F} - 32) \times 5/9 = T_{°C}$

Temperature: $(T_{°C} \times 9/5) + 32 = T_{°F}$

MONOPOLE IMPEDANCE AND GAIN MEASUREMENTS OF FINITE GROUND PLANES

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The purpose of the work described in this report is to determine if it is possible to make acceptably accurate input impedance and gain measurements of monopoles on a reduced size ground plane. Ideally, monopoles are measured with the antenna located on an infinite, perfectly conducting, ground plane. Practically, measurements are made on a test site with dimensions largely determined by the cost and availability of the space occupied by the site. Theoretical calculations show that the radius of a highly conducting ground plane should be at least 2 λ, where λ = wavelength, for measuring the input impedance of 0.25 λ monopoles. At 25 MHz, the lowest frequency considered here, such a ground plane would require a space at least 48 m (157 ft) in diameter. Model impedance measurements and calculations presented in this report imply that a space on the order of 10 by 11 m (33 by 36 ft) may be sufficient if the researcher uses 16 resistively-loaded wire radials to extend a 3.66 by 4.88 m (12 by 16 ft) solid metal ground plane. Measured insertion loss data acquired using a 1:5 scale model ground plane with resistively-loaded radials indicate that this area is sufficiently large for gain measurements as well. Measured and calculated monopole SWR and insertion loss on a full-scale ground plane verify the results of the model measurements.

Key words: gain measurement; impedance measurement; insertion loss; monopole antenna; reduced size ground plane; standard antenna; standing wave ratio.

1. INTRODUCTION

A gain-standard antenna, or simply standard antenna, is one having gain-pattern and input-impedance characteristics that are known with a small degree of uncertainty. Standard antennas are typically used in gain-transfer measurements to determine the gain of test antennas by comparing the signal level received by the test antenna to that received by the standard antenna when it replaces the test antenna [in a plane-wave field generated] on an antenna test range. For simplicity and accuracy, the standard antenna should have polarization and gain patterns similar to those of the test antenna. Therefore, a monopole standard antenna should be used to determine the gain of a monopole test antenna if the standard's input impedance and gain are to be accurately determined.

Accurately determining gain and impedance parameters for standard monopoles ordinarily requires that they be located on an electrically large, highly conducting ground plane [1,2][1]. The primary contribution of this report is applying a resistive loading technique [3] to make a small ground plane appear to be electrically larger than its physical dimensions would indicate.

2. MODEL MEASUREMENTS

2.1 Model Ground Planes

Model ground planes were centered on a 2.14 m (7.02 ft) diameter foamed polystyrene disk on top of a cart, placing the feedpoints 1.6 m (5.25 ft) above ground, on the underside of the ground plane. The relative dielectric constant of the disk is 1.03 to approximate free space. All measurements were made with a computer-controlled network analyzer system located on the cart below the center of this disk, thus minimizing cable lengths between the instrumentation and the antenna feedpoints.

In this report, the uncertainty of the standard monopole input impedance was determined by comparing measured and calculated impedance data for various ground plane configurations. The uncertainty of the standard monopole gain was determined by comparing measured and calculated insertion loss data for two

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* Electromagnetic Fields Division, Center for Electronics and Electrical Engineering, National Engineering Laboratory.

[1] Numbers in brackets refer to references in section 4.

identical monopoles at specified separation distances on a ground plane having resistively-loaded radials and designed to satisfy the input impedance requirements. Minimizing the difference between measured and calculated input impedance was assumed to be the most stringent design criterion for the ground plane. After the ground plane configuration was determined based on impedance measurements, the insertion loss data were measured.

2.2 Using Input Impedance to Determine Minimum Circular Ground Plane Size

Input impedance of thin linear monopoles was calculated by extrapolation from the data tables in [4], supplemented by data tables calculated using equations in [5]. The input impedance of thin linear monopoles over finite ground planes was obtained from the data in [6], "for monopole antenna[s] at the center of a perfectly conducting circular disk in free space."

Figures 1, 2, and 3 show the results of the initial set of impedance versus ground plane disk radius measurements performed with a single, fixed length, monopole chosen to duplicate that used for Richmond's

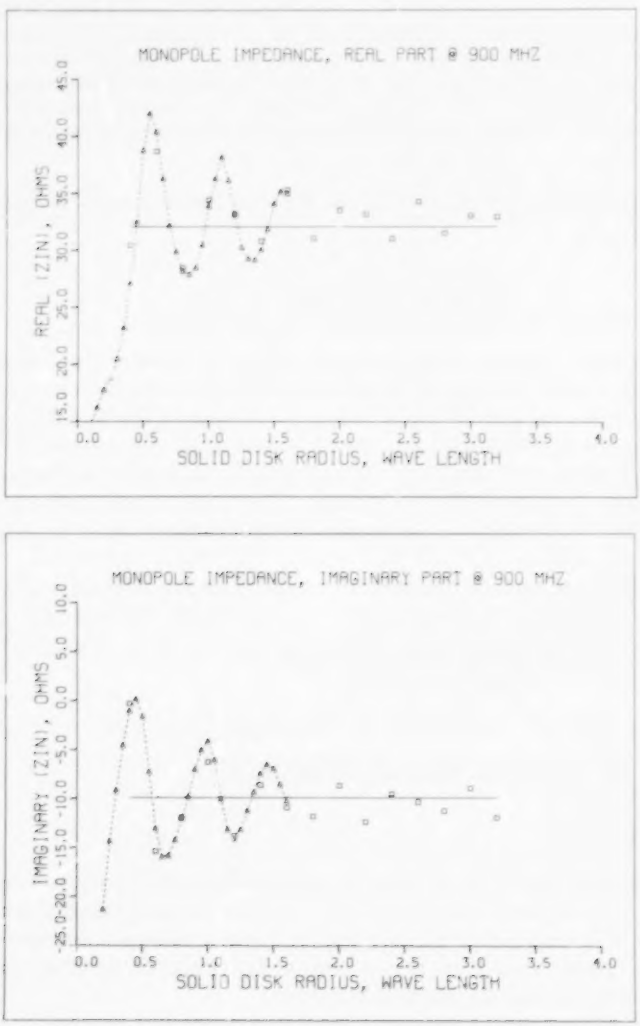
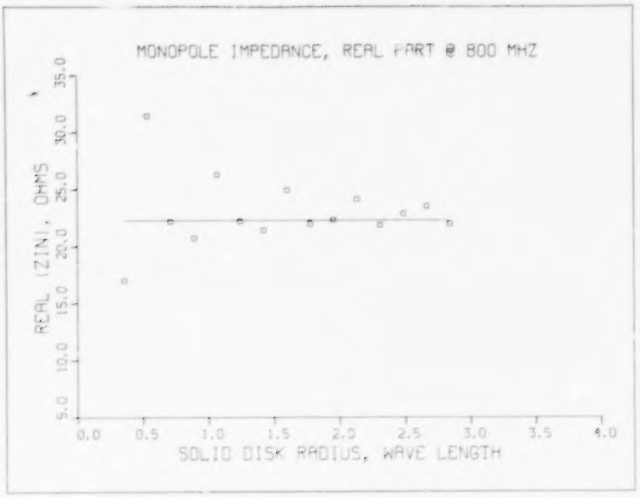

MONOPOLE IMPEDANCE, REAL PART @ 900 MHZ
REAL (ZIN), OHMS
15.0 20.0 25.0 30.0 35.0 40.0 45.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
SOLID DISK RADIUS, WAVE LENGTH
MONOPOLE IMPEDANCE, IMAGINARY PART @ 900 MHZ
IMAGINARY (ZIN), OHMS
-25.0 -20.0 -15.0 -10.0 -5.0 0.0 5.0 10.0
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
SOLID DISK RADIUS, WAVE LENGTH

FIGURE 1. *Measured and calculated input impedance versus solid disk radius for a 0.0763-m monopole at 900 MHz.*
Calculated by Richmond —Δ—Δ—.
Calculated by King et al. _____ (infinite disk radius).
Measured □ □ □ □.



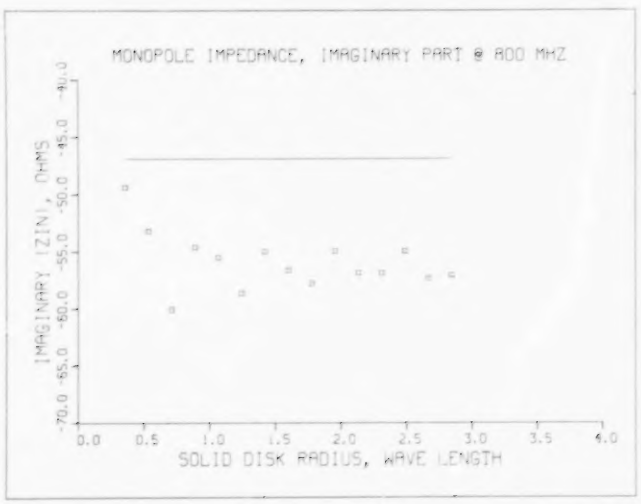


FIGURE 2. *Measured and calculated input impedance versus solid disk radius for a 0.0763-m monopole at 800 MHz.*
Calculated by King et al. _____ *(infinite disk radius).*
Measured □ □ □ □.

calculations at 900 MHz [6]. Monopole length and radius were 0.229 λ (0.0763 m) and 0.003 λ (0.001 m) respectively. The ground plane was a 0.18 cm thick aluminum disk having a 0.4 λ (0.133 m) radius, centered on a sheet of aluminum foil lightly glued to the "free-space" disk. Input impedance was measured at 800, 900, and 1000 MHz using the following procedure. The overall diameter of the ground plane was reduced in size by removing successive rings of aluminum foil 0.2 λ (0.067 m) wide until only the 0.4 λ (0.133 m) radius disk remained. The measurements were made each time the annulus of foil was removed. Figures 4 and 5 show the foil covered disk at the start and midpoint of the measurement. Figure 6 shows the computer-controlled network analyzer located beneath the disk.

Excellent agreement between the data sets shown in figure 1 provides a high degree of confidence in the measured data, shown by the squares, and in the calculated data, both for an infinite radius ground plane, shown by the solid lines, and for finite radius ground planes in free space, shown by dashed lines connecting the triangles. Figures 2 and 3 show that the imaginary parts of the input impedance data do not converge to the solid lines as do those in figure 1. The measured or calculated data may not be as accurate for nonresonant monopoles as for resonant monopoles. Resonance is the frequency at which the imaginary part of the input impedance is zero for a monopole with a length near 0.25 λ. For this test monopole, resonance occurs at 940 MHz.

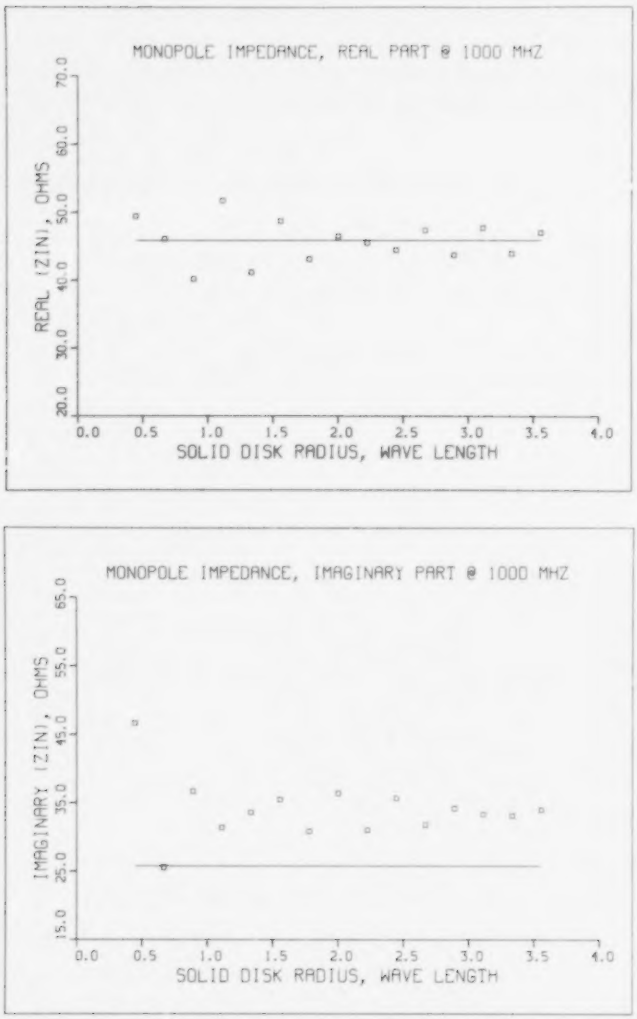


FIGURE 3. *Measured and calculated input impedance versus solid disk radius for a 0.0763-m monopole at 1000 MHz.*
Calculated by King et al. _____ *(infinite disk radius).*
Measured □ □ □ □.

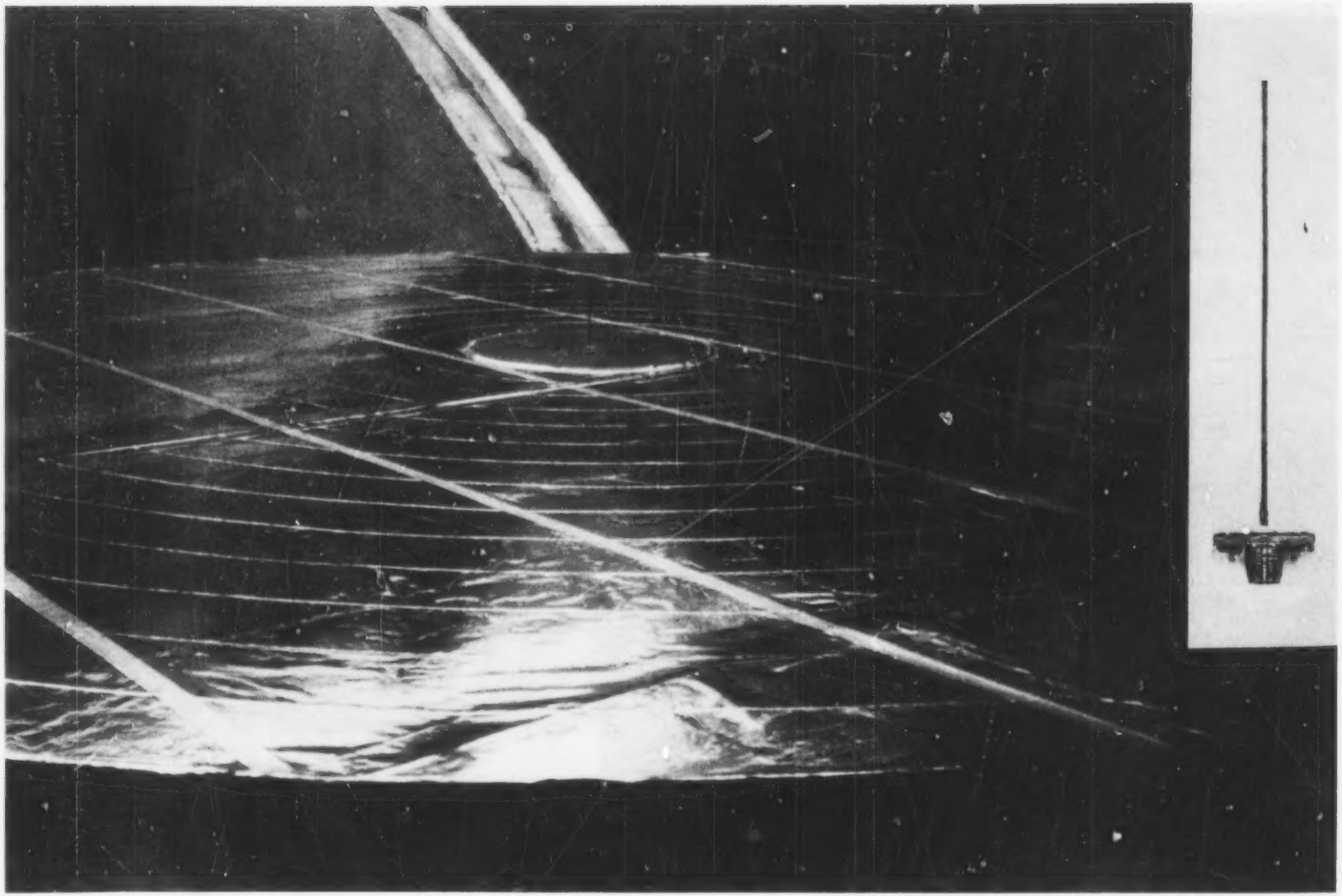


FIGURE 4. *View looking down on the 0.0763-m monopole centered on the inclined solid disk. The monopole element is shown full scale in the inset contact print.*

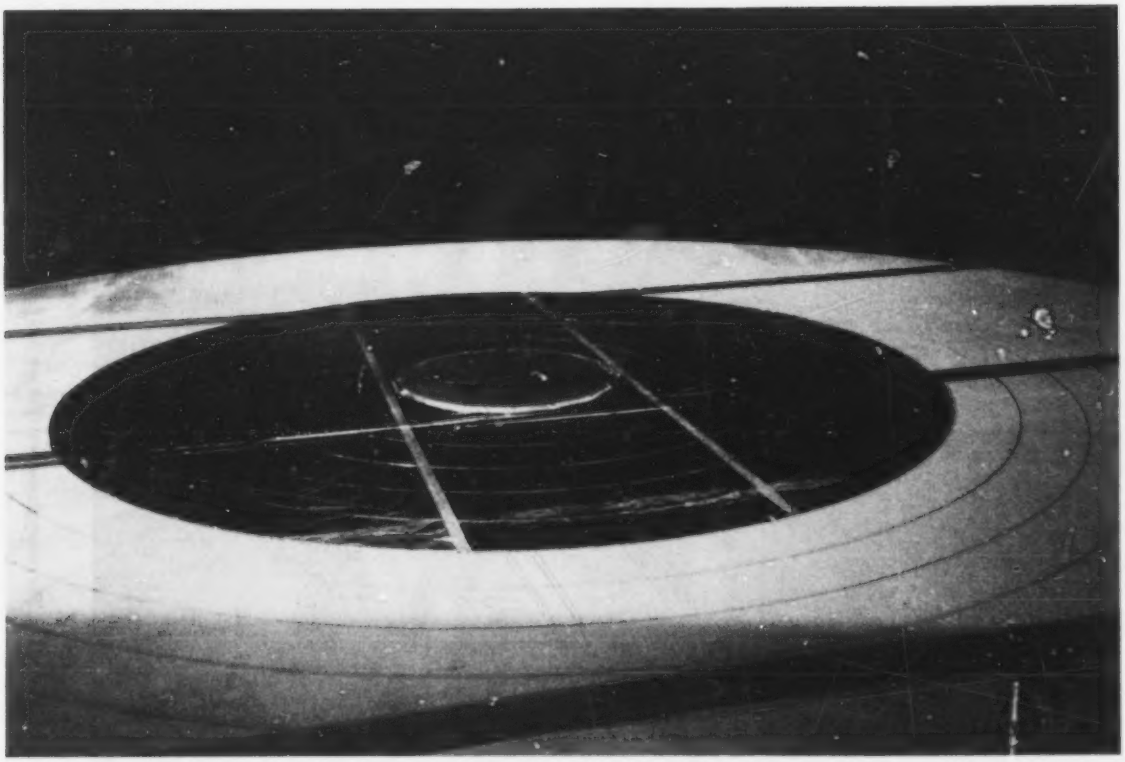


FIGURE 5. *View of the solid disk with eight annuli removed.*

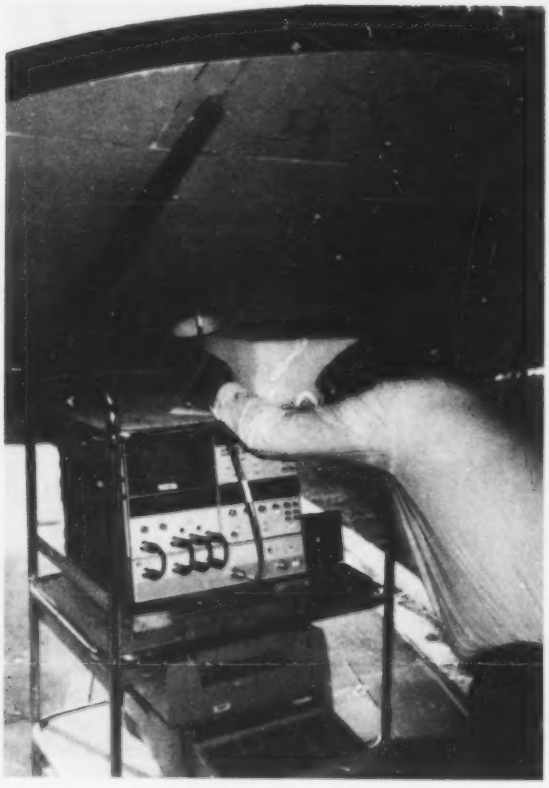


FIGURE 6. *Underside of the disk showing instrumentation used for the measurements.*

The second set of measurements determined how well wire radials simulate a solid disk for impedance measurements. It was a duplicate of the first measurement except the aluminum foil ground plane was replaced by 16 evenly spaced #26 copper wire radials as shown in figure 7. Impedance was measured, a 0.2 λ length was clipped from the end of each radial, and the measurements were repeated until only the 0.4 λ

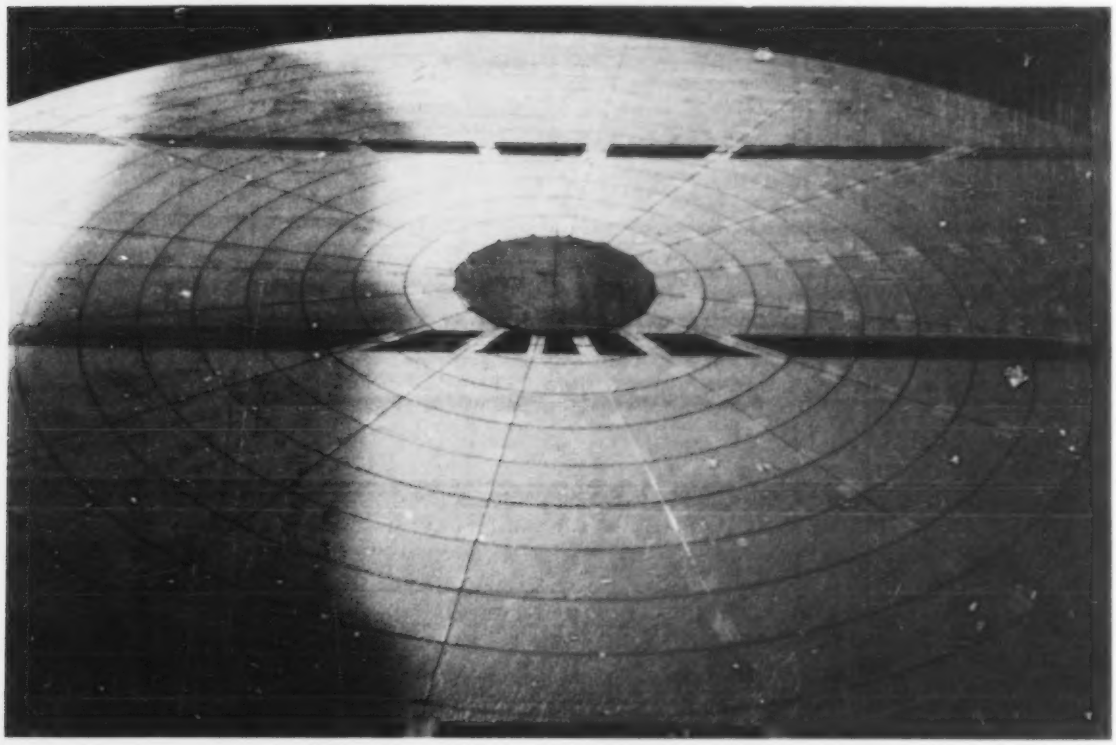


FIGURE 7. *View of the wire disk model with 16 evenly-spaced wire radials.*

radius disk remained. Figures 8, 9, and 10 differ substantially from the previous figures, and the data show no indication of convergence to the infinite radius, solid disk line. The choice of 16 radials is assumed to be a reasonable number that would be acceptable for use by industry personnel to fabricate temporary or permanent monopole test facilities.

The third set of measurements determined if Altshuler's resistively-loading technique [3] would work for this particular application. This technique alters the standing-wave current distribution on a linear wire antenna so that it is more like a traveling-wave distribution by inserting a 240-Ω resistor 0.25 λ from the open end of the wire. The resistor tends to terminate the wire in its characteristic impedance by minimizing the wave reflected at the wire end. The result is a large traveling wave with a small, superimposed standing wave on the wire. As viewed from the monopole terminals, the ground plane with resistively-loaded radials appears to be very large.

Eleven 240-Ω resistors, with 0.2-λ (0 083-m) spacings starting at the open end of the wire, were placed serially in parallel with each wire radial. The wires bridging the outermost resistors were cut away. Monopole impedance was measured, then the outermost resistors and 0.25-λ sections of the radials were cut away, the sections of wire bridging the next outermost resistors were cut away, and the measurements were repeated until only the 0.4-λ radius disk remained. Figures 11, 12, and 13 show a considerable improvement in the behavior of the input impedance as a function of radial length, indicating that resistively-loaded radials do work for this particular application.

At this point, model measurements closely resembling the proposed gain and impedance measurements are performed using a resistively-loaded ground plane.

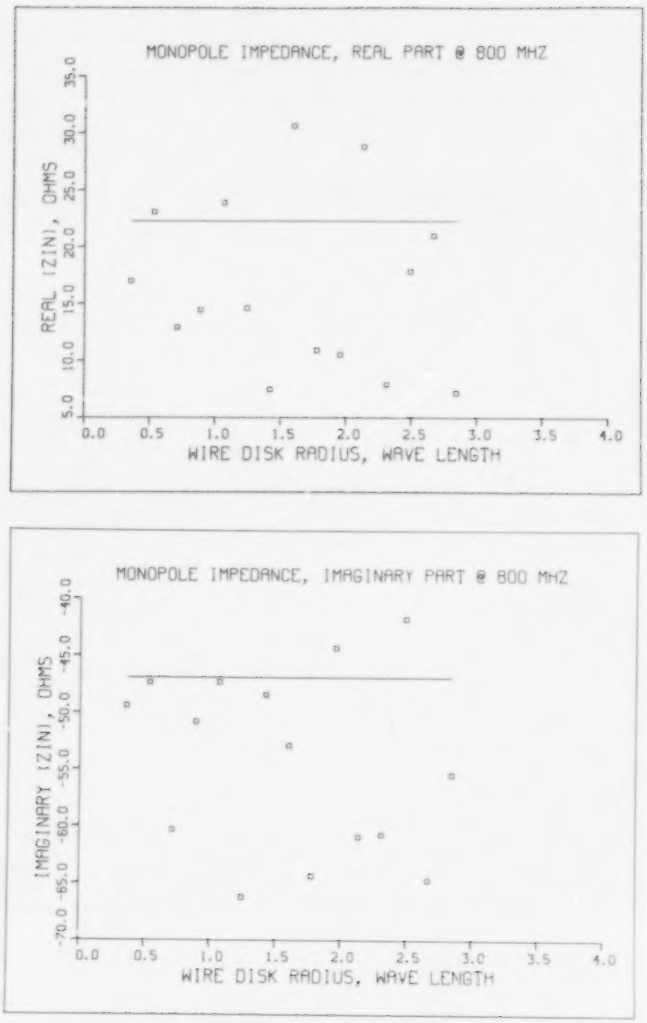
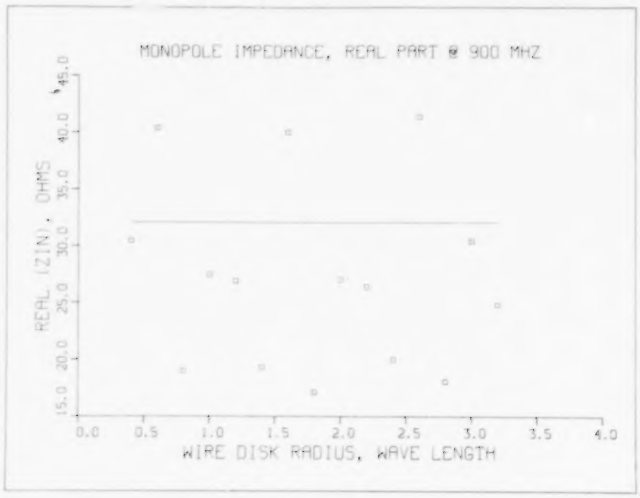


FIGURE 8. *Measured and calculated input impedance versus wire disk radius for a 0.0763-m monopole at 800 MHz.*
Calculated by King et al. _____ *(infinite disk radius).*
Measured □ □ □ □.



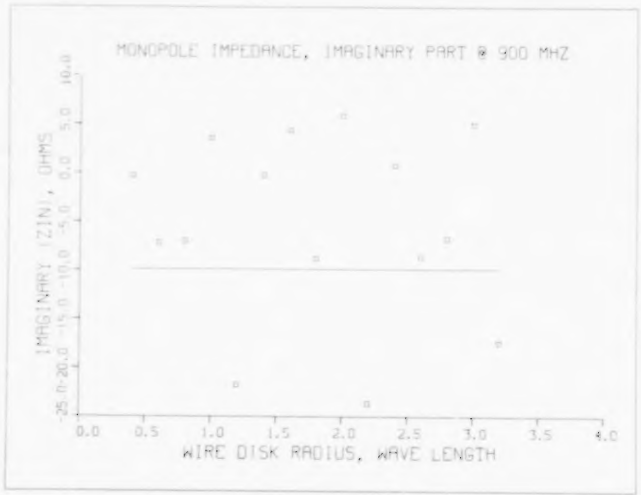


FIGURE 9. *Measured and calculated input impedance versus wire disk radius for a 0.0763-m monopole at 900 MHz. Calculated by King et al.* _____ *(infinite disk radius). Measured* □ □ □ □.

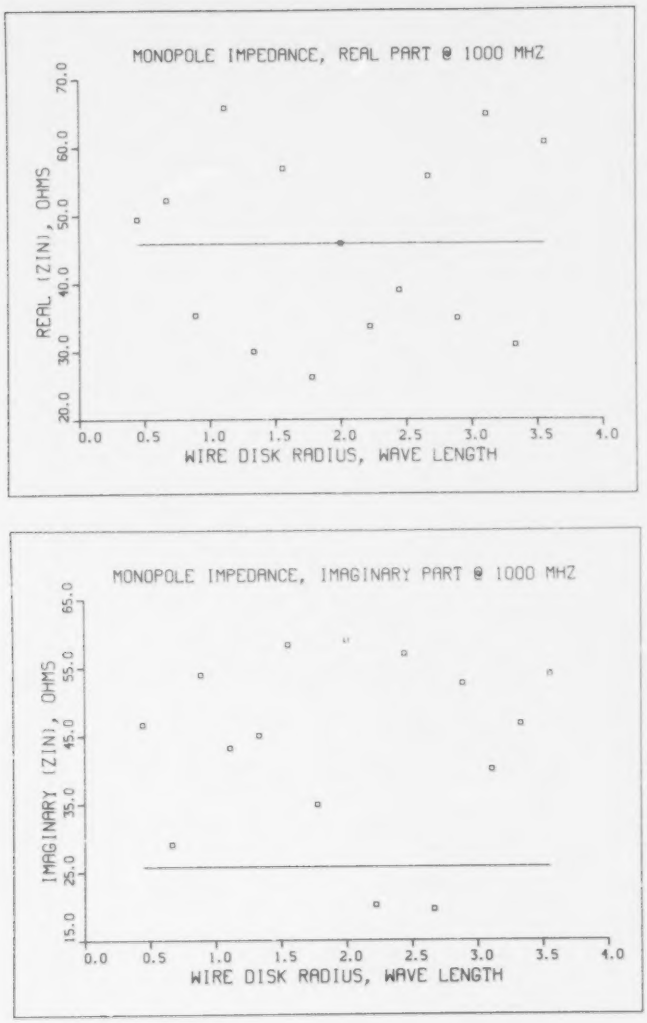


FIGURE 10. *Measured and calculated input impedance versus wire disk radius for a 0.0763-m monopole at 1000 MHz.*
Calculated by King et al. _____ *(infinite disk radius).*
Measured □ □ □ □.

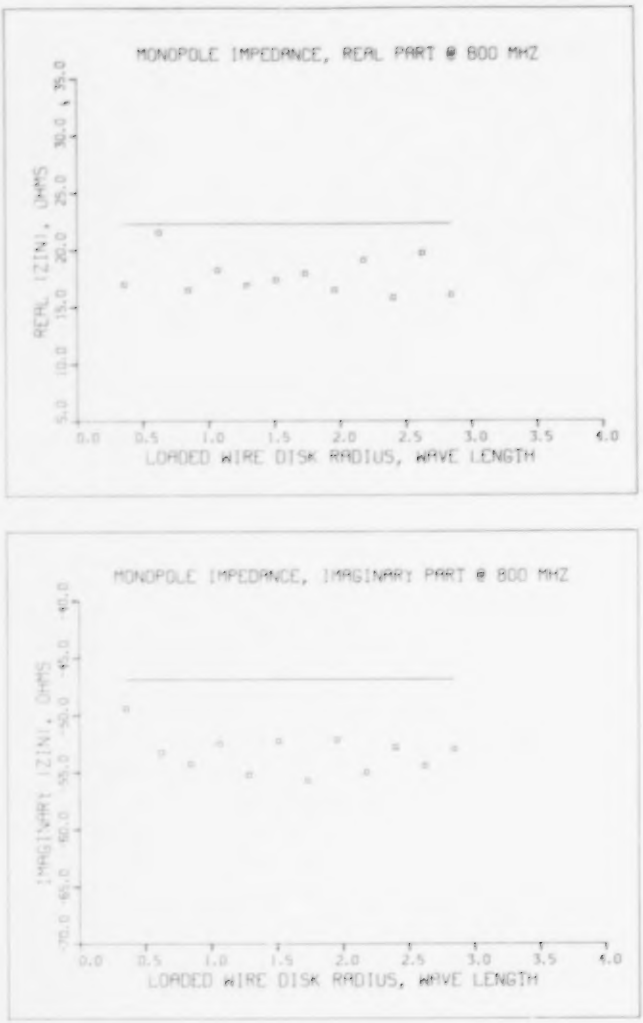


FIGURE 11. *Measured and calculated input impedance versus loaded wire disk radius for a 0.0763-m monopole at 800 MHz.*
Calculated by King et al. _____ *(infinite disk radius).*
Measured □ □ □ □.

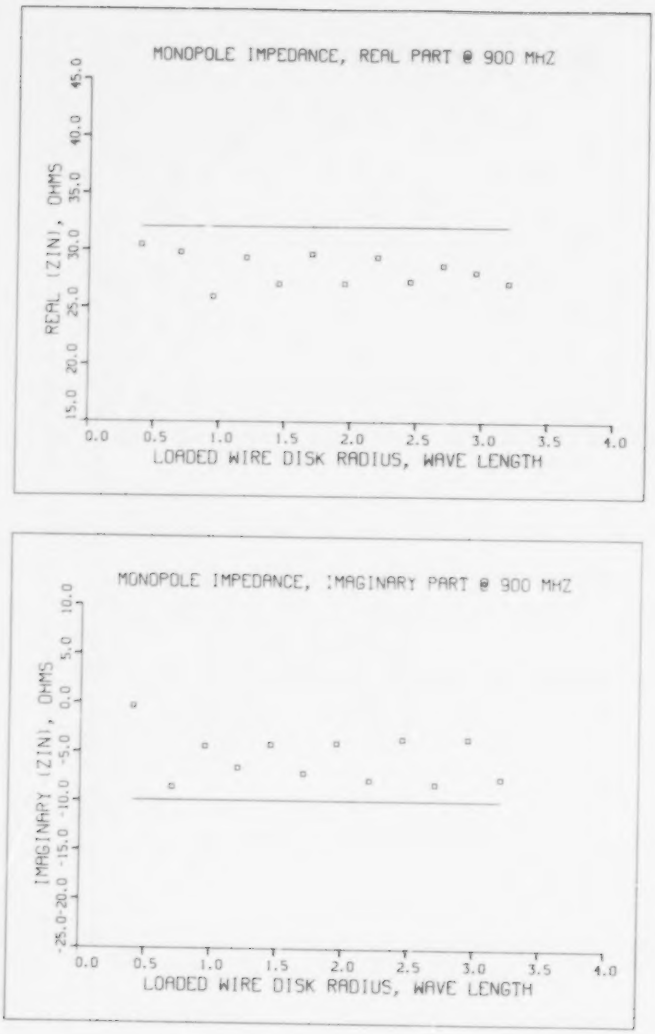
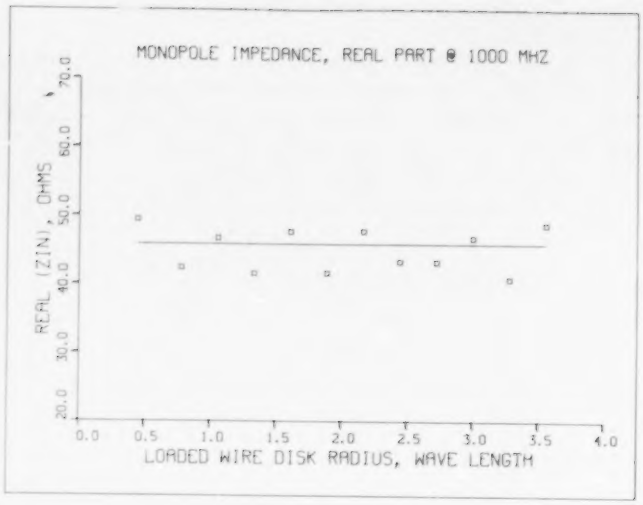


FIGURE 12. *Measured and calculated input impedance versus loaded wire disk radius for a 0.00763-m monopole at 900 MHz.*
Calculated by King et al. _____ *(infinite disk radius).*
Measured □ □ □ □.



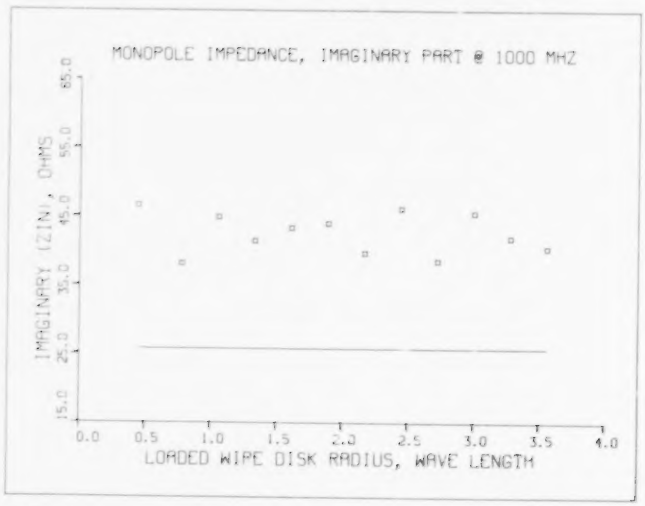


FIGURE 13. *Measured and calculated input impedance versus loaded wire disk radius for a 0.0763-m monopole at 1000 MHz.*
Calculated by King et al. _____ *(infinite disk radius).*
Measured □ □ □ □.

2.3 Model Ground Plane Configuration and Verification

Based on data in figures 11, 12, and 13, a model ground plane was fabricated assuming a 1:5 scale factor at the lowest design frequency of 30 MHz. The result is an aluminum sheet 0.4 by 0.6 λ (0.8 by 1.2 m) at 150 MHz with 16 240-Ω resistors soldered to brass screws located at equal angular spacings around the edge of the sheet. No. 26 wire radials, equal to the monopole length, were soldered to these resistors as shown in figure 14. Table 1 lists the pertinent data for the model configuration shown in figure 15 with the 0.424 m (1.39 ft) monopoles at the 0.4 m (1.3 ft) separation distance.

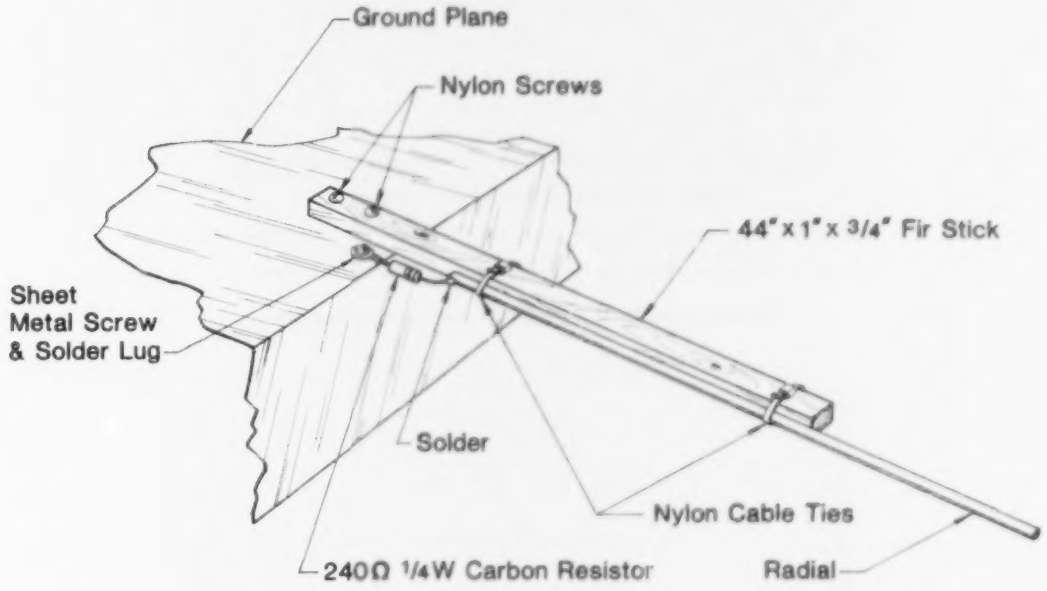


FIGURE 14. *Attachment of the 240-Ω resistor between the ground plane and the wire radials.*

TABLE 1. *Test frequencies and monopole dimensions for the model ground plane.*

Full scale frequency, MHz	*Scale factor*	*Actual measured frequency band, MHz*	*Monopole length[a], m*	*Monopole radius, cm*	*Resonant frequency, MHz*
25–50	1:5	125–225[b]	0.424	0.118	168
150–174	1:5	750–870	0.093	0.118	731
400–512	1:1	400–520	0.166	0.118	420
800–960	1:1	800–960	0.086	0.118	790

[a] These lengths are the geometric means of the free-space values of λ/4 for the two frequencies listed in column 1, with the scale factor applied.
[b] Two monopoles would have been a better choice for this wide band. Emphasis was placed on the more difficult, lower end of this band.

Model ground plane performance verification consisted of measuring input impedance (derived from the reflection coefficient) of a single monopole centered on the ground plane extended with resistively-loaded wire radials equal to the monopole length. Two identical monopoles were then placed 0.4 m (1.3 ft) apart on the ground plane's longer center line, and the insertion loss (obtained directly as the magnitude of the transmission coefficient) was measured. This measurement was repeated at an 0.8 m (2.6 ft) separation distance. Below 150 MHz, the 0.4 m (1.3 ft) distance is close to the $\lambda/2\pi$ distance from an electrically small monopole to the boundary between its reactive near-field and far-field regions. Far-field illumination conditions normally imposed for antenna tests are not required for the insertion loss because it is based on mutual impedance calculations requiring only a sinusoidal current distribution on the antennas. Since the gain

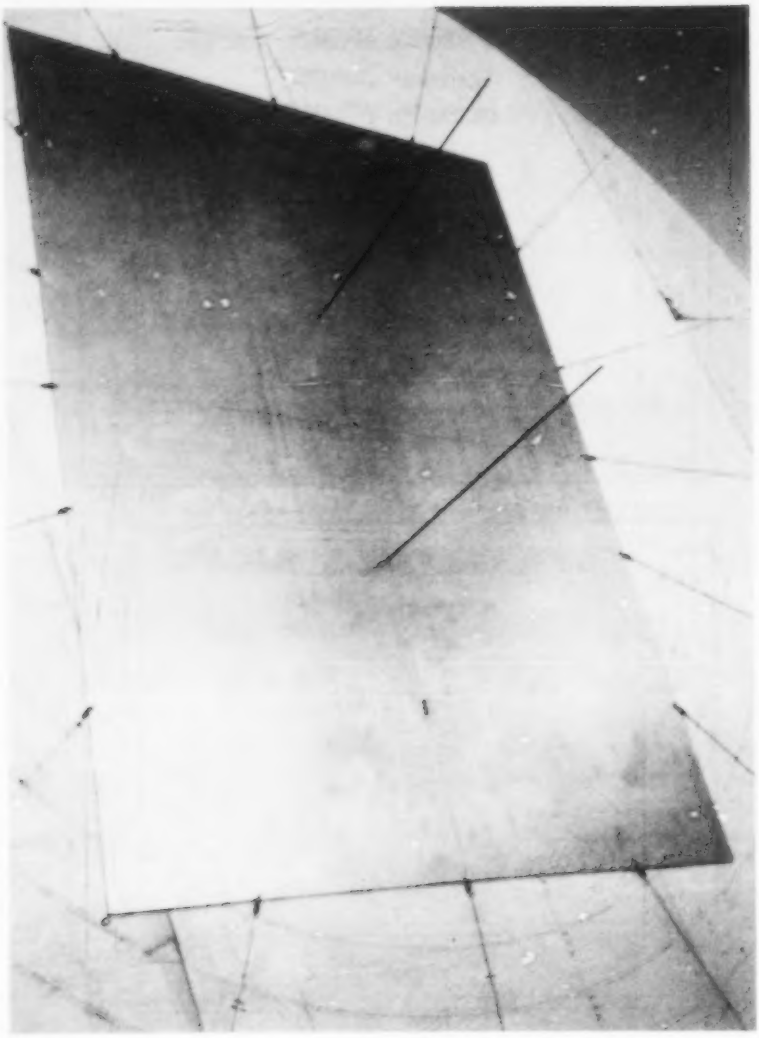
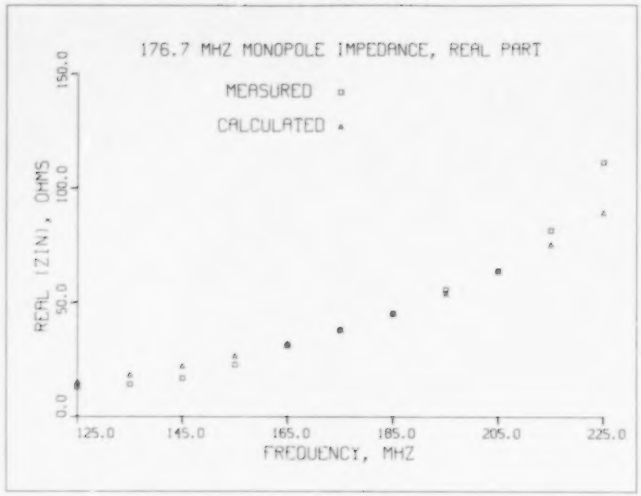


FIGURE 15. *View of the model ground plane with the 0.424-m monopoles spaced 0.4 m apart. The 16 resistors and wire radials are evenly spaced in the angle around the edge of the rectangular aluminum sheet.*

product of two antennas is an intrinsic part of insertion loss at far-field distances, gain of a test monopole relative to that of a standard monopole of similar length may be measured at near-field distances with an uncertainty based on the results of the insertion loss data.

Using a single monopole and loaded wire radial length for each set of measurements in a particular frequency band provides measured and calculated data indicating the bandwidth over which these measurements are valid. That is, the monopole and radials are electrically shorter below the monopole design frequency and longer above it. Measured and calculated input impedance versus frequency data are presented in figures 16 through 19. Agreement between these two sets of data is good, especially near the resonant frequency of the monopole, for all but the highest frequency band. Apparently the electrical spacing between the radials at the edge of the ground plane is too great, and reflections from the edge are occurring. This is shown in data in the next section by the cyclical variations in measured insertion loss data for this frequency band.

Voltage standing wave ratio, or SWR, is a scalar quantity related to input impedance that is commonly used to specify antenna performance. Figures 20 and 21 show SWR versus frequency, calculated using the impedance data from the preceding four figures. The agreement between measured and calculated data is quite good.



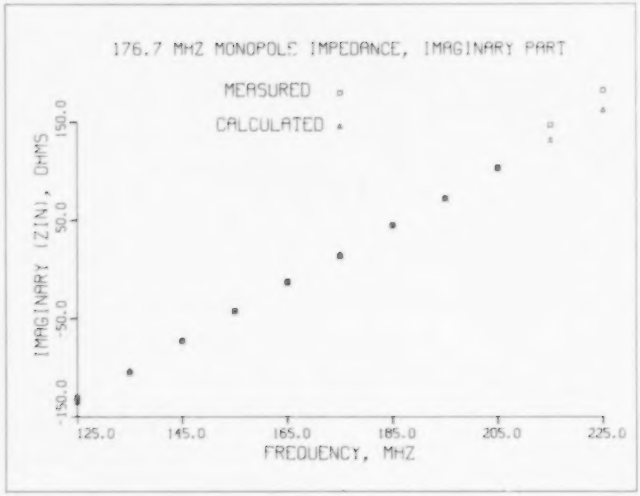
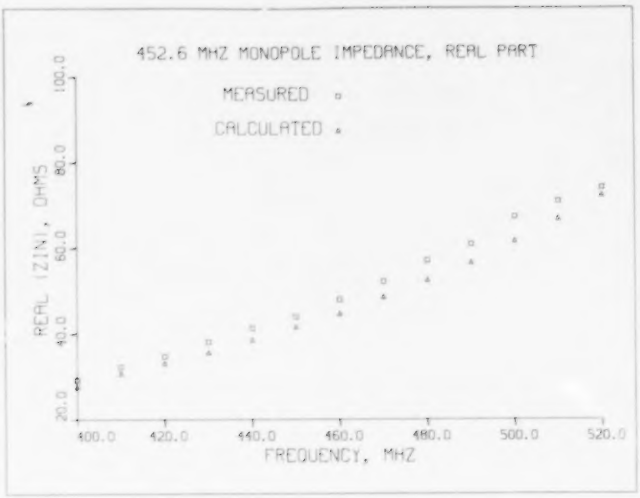


FIGURE 16. *Input impedance versus frequency for the 0.424-m monopole on the model ground plane.*



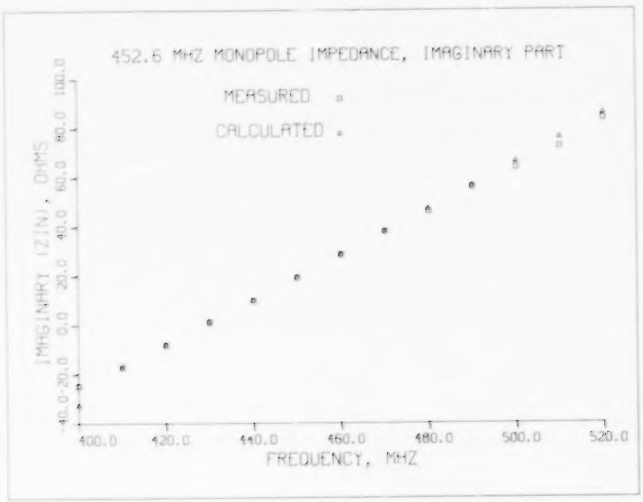


FIGURE 17. *Input impedance versus frequency for the 0.166-m monopole on the model ground plane.*

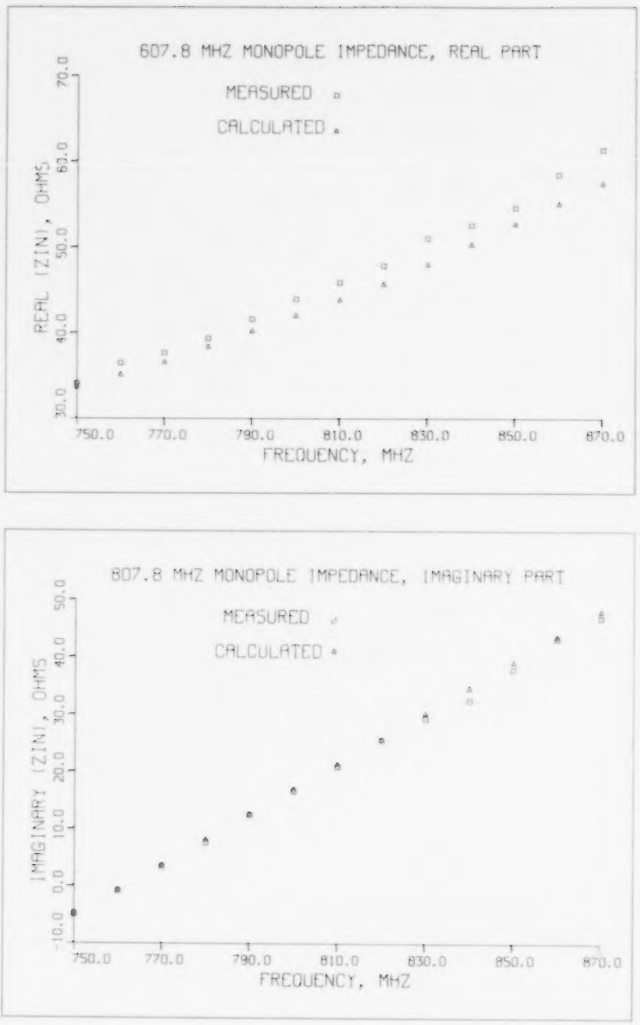


FIGURE 18. *Input impedance versus frequency for the 0.093-m monopole on the model ground plane.*

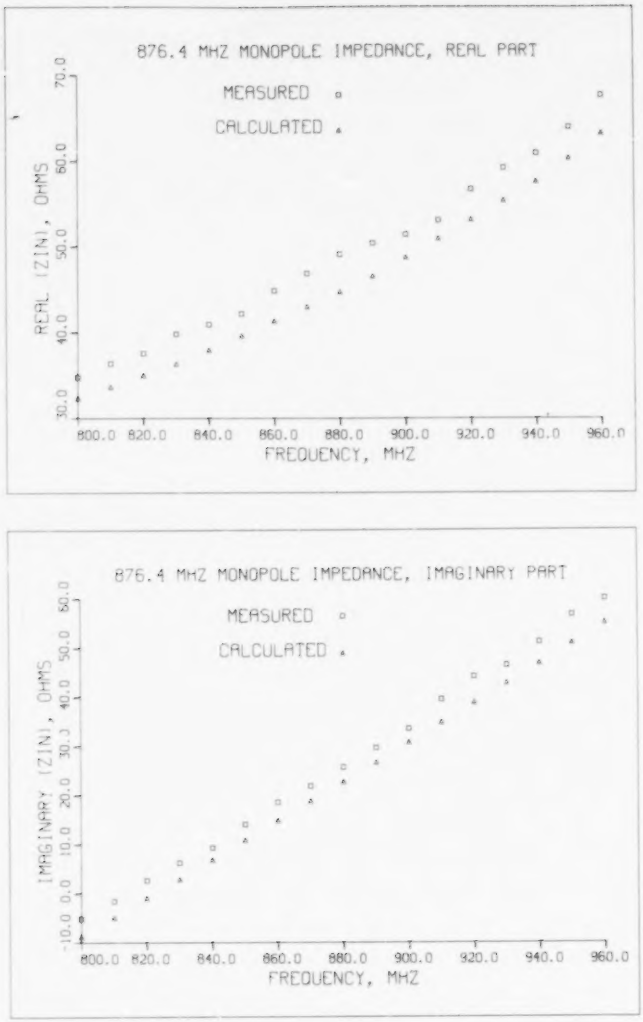


FIGURE 19. *Input impedance versus frequency for the 0.086-m monopole on the model ground plane.*

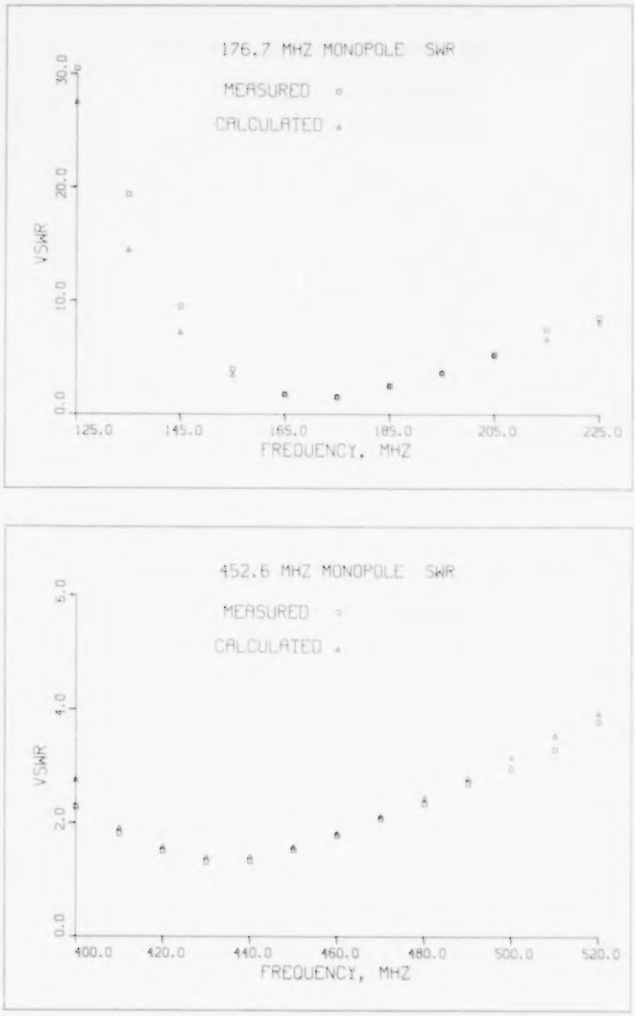


FIGURE 20. *Input SWR versus frequency for the 0.424-m and 0.166-m monopoles on the model ground plane.*

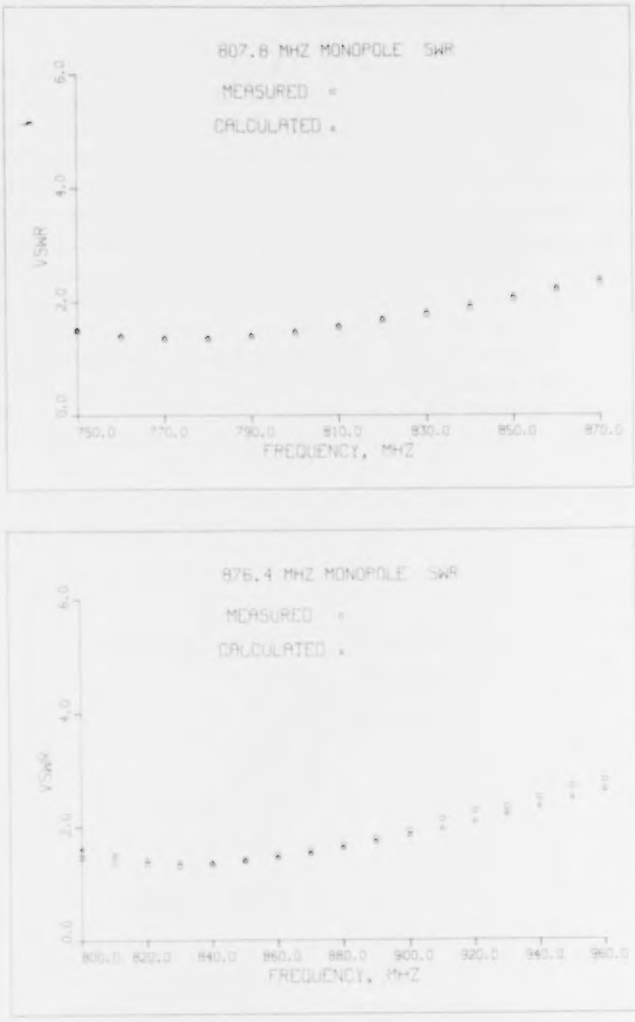


FIGURE 21. *Input SWR versus frequency for the 0.093-m and 0.086-m monopoles on the model ground plane.*

2.4 Insertion Loss and Gain Measurements

Substitution loss [7] is a relative measure of the transmission properties of two different waveguide junctions (two-ports or two-terminal-pair networks). It is the ratio, in decibels, of the power measured in a load with the initial two-port network in place, to the power measured in the same load with the final two-port network replacing the initial one in the transmission path. It is assumed that the generator and load characteristics remain the same for the two measurements. Insertion loss, a special case of substitution loss, is measured when the initial two-port network is perfect (a lossless, reflectionless, waveguide junction), and the reflection coefficients of the generator and load, ρ_G and ρ_L, are not necessarily equal to zero. The network analyzer measures the insertion loss between the terminals of two monopoles on the ground plane directly as the transmission coefficient. Initial load power is measured by the analyzer during self-calibration.

Attenuation, another special case of substitution loss, is measured when the initial two-port network is perfect and $\rho_G = \rho_L$ as well. It is assumed that the difference between the transmission coefficient and attenuation is accounted for in the error budget of the network analyzer. Insertion loss data was calculated using the computer program in appendix A based on the equations in NBS Technical Note 1089 [8] assuming $\rho_G = \rho_L$, and is given as a *positive* number. To avoid confusion, the term insertion loss rather than attenuation is used in this document.

Figures 22–25 show measured and calculated insertion loss versus frequency for two identical monopoles at two separation distances. The effects of the ground plane edges are shown by the cyclical variation of

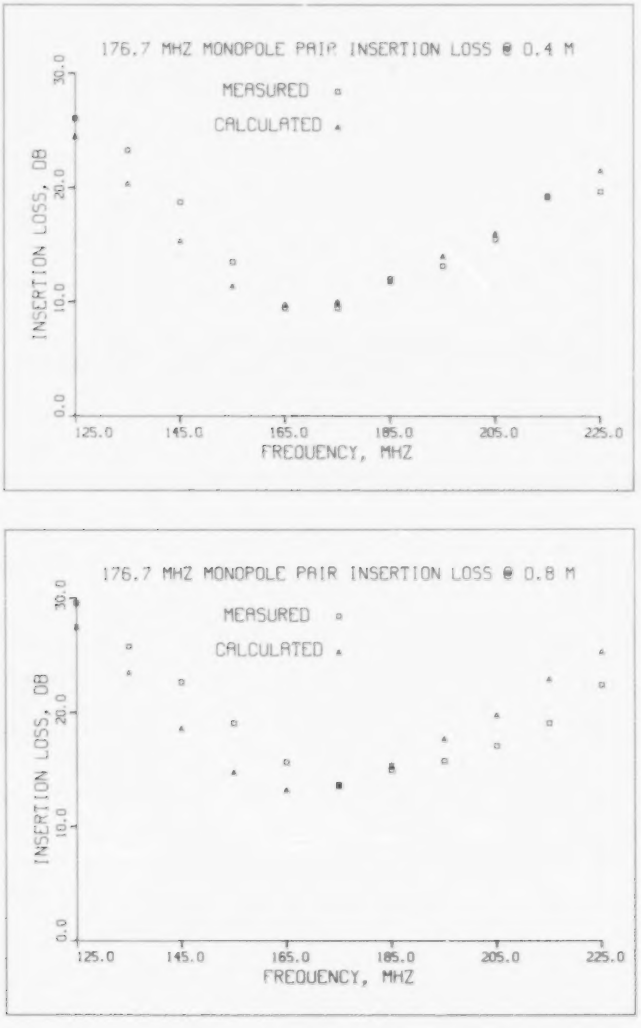


FIGURE 22. *Insertion loss versus frequency for the 0.424-m monopole at 0.4-m and 0.8-m separation distances.*

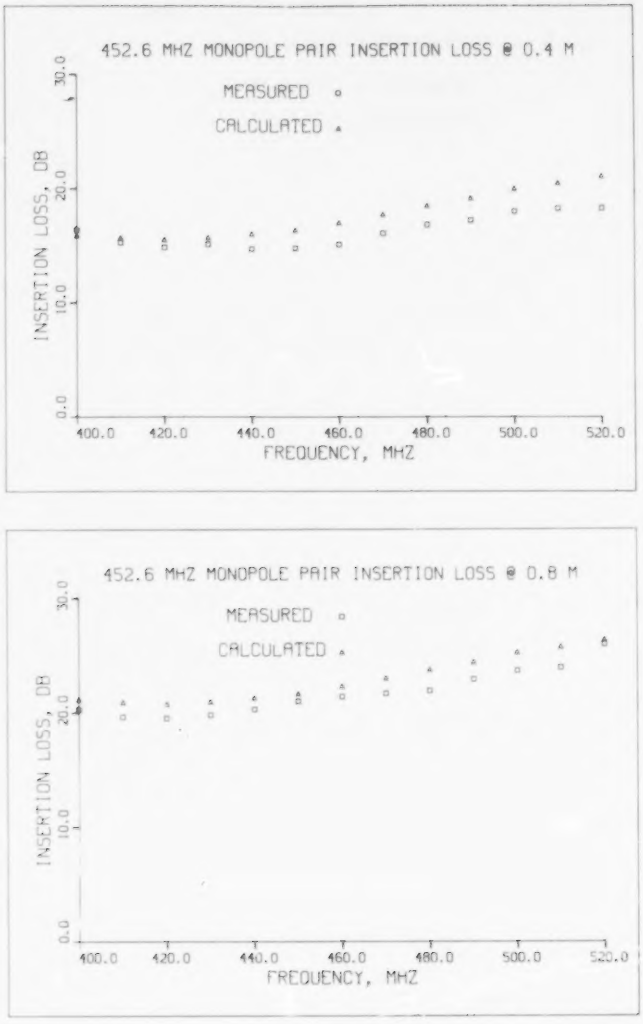
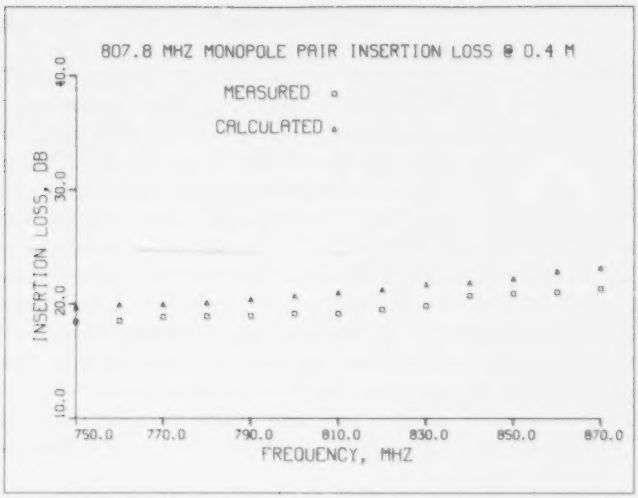


FIGURE 23. *Insertion loss versus frequency for the 0.166-m monopole at 0.4-m and 0.8-m separation distances.*

insertion loss with frequency that is most apparent at the two higher frequency bands. Generally, the 0.4 m (1.3 ft) separation distance results in a smaller difference between measured and calculated data. Note that insertion loss includes mismatch losses at the monopole terminals as well as the path loss between the monopoles. One-half of the difference between the measured and calculated data is a good estimate of the accuracy of realized gain of one of the identical monopoles assuming the ground plane is perfect. Realized gain is "The power gain of an antenna in its environment, reduced by the losses due to the mismatch of the antenna input impedance to a specified impedance" [9]. A 50-Ω specified impedance is assumed for all measured and calculated data in this document.



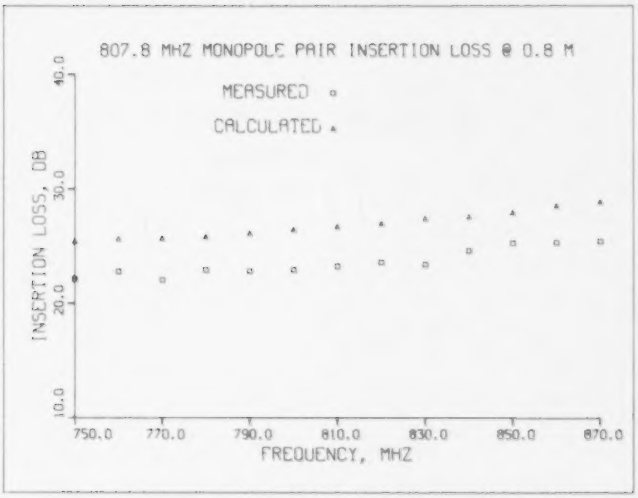


FIGURE 24. *Insertion loss versus frequency for the 0.093-m monopole at 0.4-m and 0.8-m separation distances.*

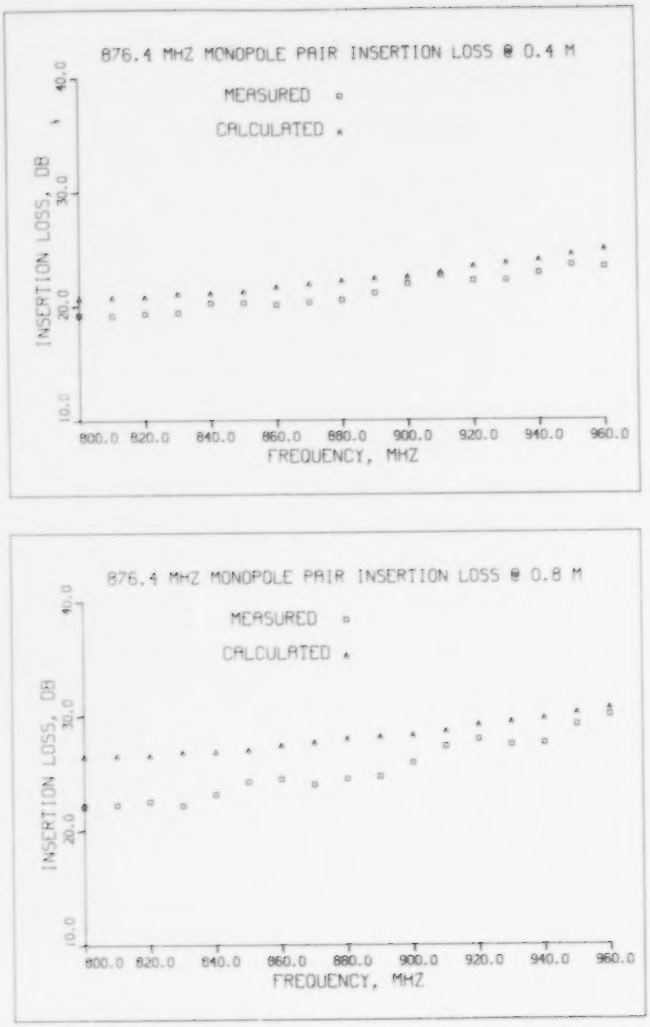


FIGURE 25. *Insertion loss versus frequency for the 0.086-m monopole at 0.4-m and 0.8-m separation distances.*

Realized gains for the model monopoles are presented in table 2. They were calculated using computer program GAIN77 in appendix B based on equations in NBS Technical Note 1098 [10]. Mismatch losses were subtracted from the power gains to obtain the realized gains. Mismatch losses versus frequency, calculated using the SWR data in figures 20 and 21, are presented in figures 26 and 27.

As can be seen in table 2, the realized gains of these monopoles are only slightly below the theoretical 4.79-dB value for a very thin 0.25-λ monopole. The mismatch losses shown in figures 26 and 27 tend to be minimum near the resonant frequencies of the monopoles. Fixed frequency monopoles are usually adjusted in length to minimize this mismatch loss, although the gain changes only slightly when the length is adjusted.

TABLE 2. *Calculated realized gains for the model monopoles*[a].

Full scale frequency, MHz	*Scale factor*	*Frequency used for gain calculation, MHz*	*Monopole length, m*	*Gain relative to isotropic, dB*
35	1:5	176.7	0.424	4.54
161	1:5	807.8	0.093	4.21
452.6	1:1	452.6	0.166	4.36
876.4	1:1	876.4	0.086	4.15

[a] All monopole radii are 0.118 cm.

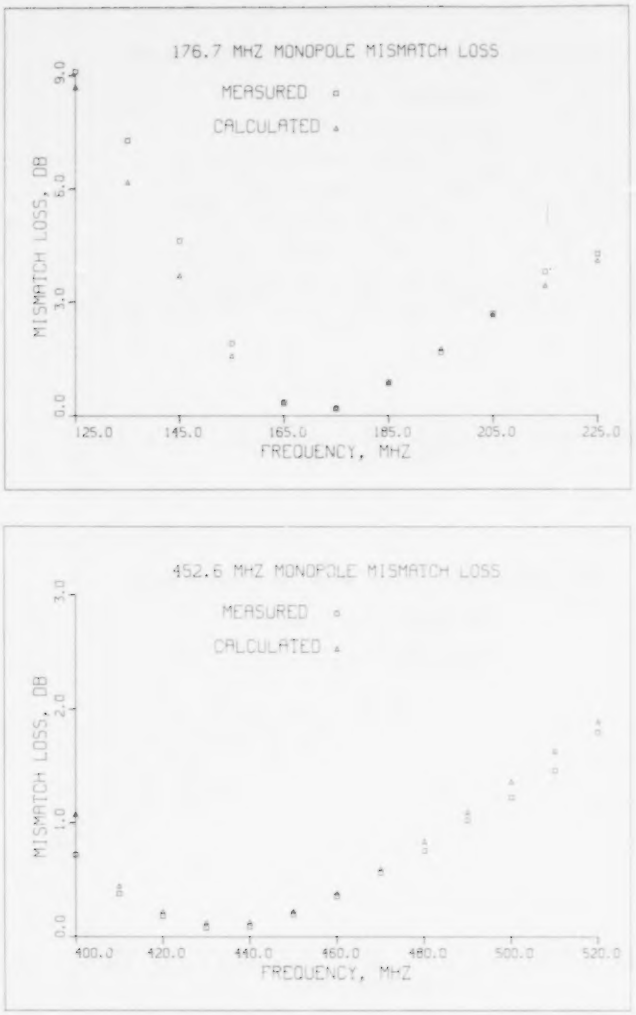


FIGURE 26. *Mismatch loss versus frequency for the 0.424-m and 0.166-m monopoles on the model ground plane.*

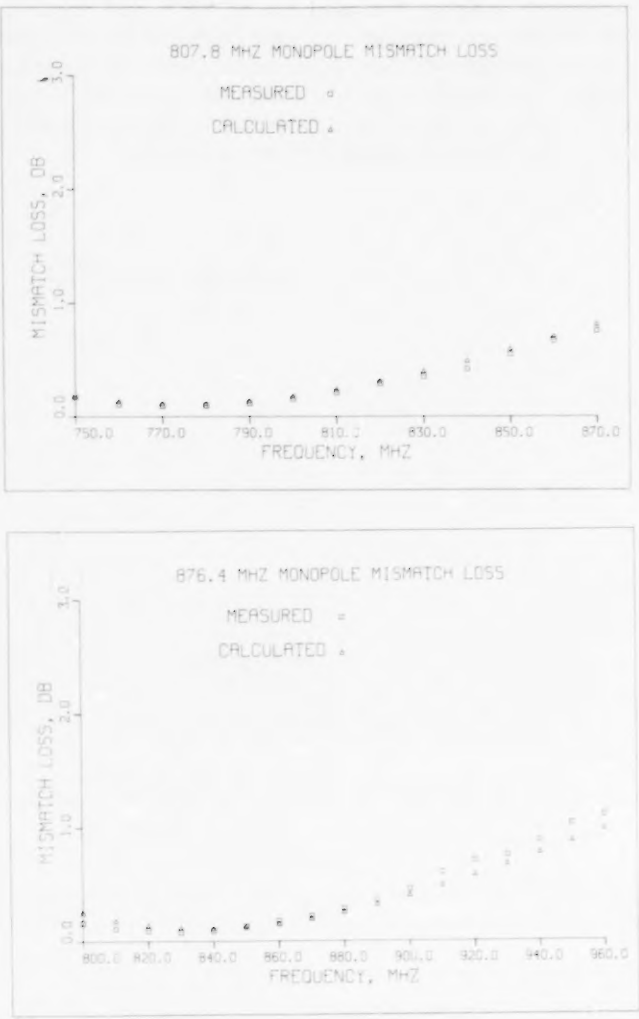


FIGURE 27. *Mismatch loss versus frequency for the 0.093-m and 0.086-m monopoles on the model ground plane.*

3. FULL-SCALE GROUND PLANE CONFIGURATION AND VERIFICATION

A 3.66 by 4.88 m (12 by 16 ft) wall-mounted microwave passive repeater forms the ground plane of the full-scale test facility. It is mounted horizontally with the attaching brackets supplied by the manufacturer. The height between the ground and the under surface of the ground plane and the feedpoints of the test antennas is 2.15 m (about 7 ft). This structure is designed to withstand 125 mph winds, and the surface flatness "Under no load . . . will be plus 0″ and minus 1/8″ where negative values indicate a concave surface," as specified by the manufacturer. A series of mounting holes for Type N panel jacks is located on a diagonal axis of the aluminum surface to allow insertion loss measurements at monopole-to-monopole separation distances of 0.06, 0.12, 0.18, 0.24, 0.36, 0.5, 1, 1.5, and 2 m. Sixteen resistively-loaded radials, having lengths equal to the monopole lengths, extend the electrical size of the ground plane at the frequencies below 175 MHz as discussed in section 2 of this document.

A computer-controlled network analyzer was used for all SWR and insertion loss measurements. Antenna SWR was measured with the test monopole mounted on the panel jack nearest the center of the ground plane. Figure 28 is a schematic representation of the ground plane and the test equipment is used for the full-scale measurements. Figure 29 is a photograph of the test facility with the 2.43 m (8 ft) monopoles and radials installed. White dielectric feedpoint assemblies may be seen in this photograph. They are used with the test antennas in the two lower frequency bands for mechanical support.

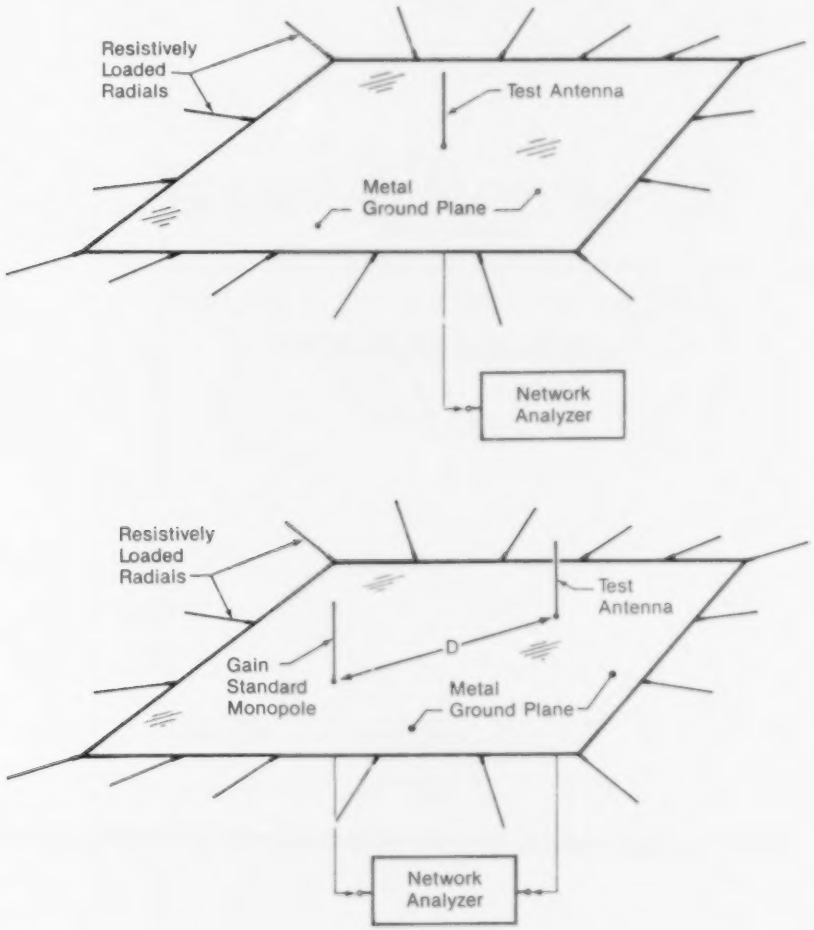


FIGURE 28. *Schematic of SWR measurement, above; schematic of insertion loss measurement, below.*

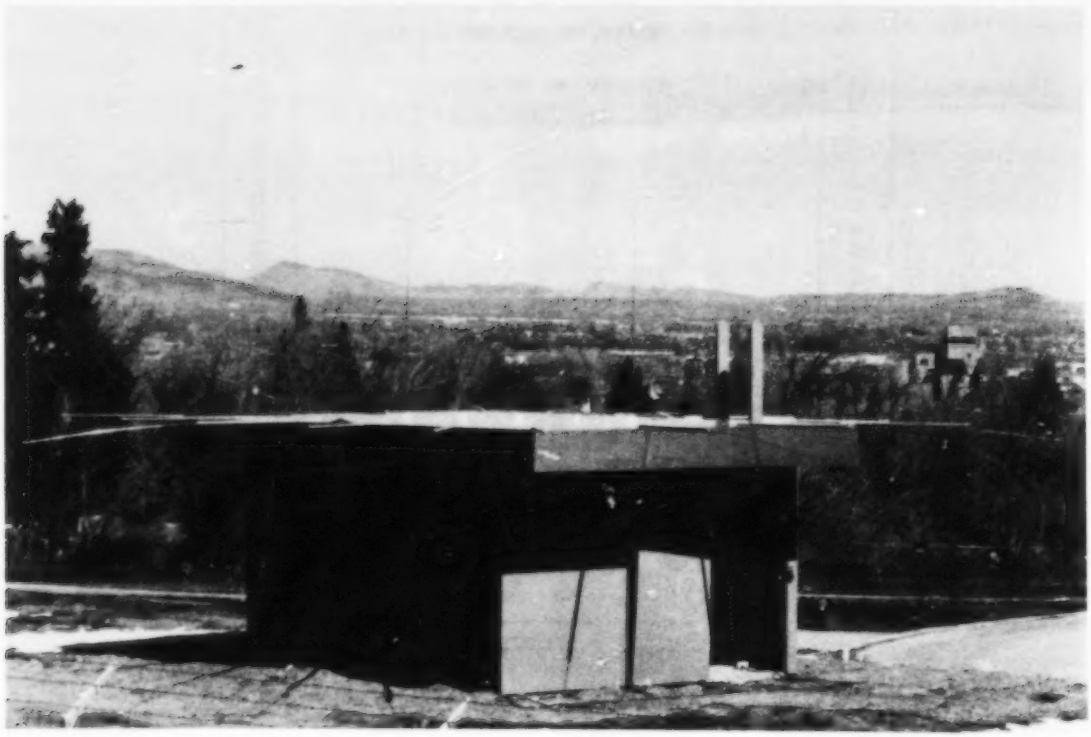


FIGURE 29. *Photograph of the full-scale test facility with 2.43-m test antennas and resistively-loaded radials.*

The characteristics of the monopole test antennas used for the full-scale measurements are given in table 3. These antennas are the gain-standard monopoles proposed for the NIJ Standard for Mobile Antennas [11].

TABLE 3. *Test frequencies and monopole dimensions for the full-scale ground plane.*

Frequency band, MHz	*Length, m*	*Base radius, c*	*Tip radius, cm*	*Resonant frequency, MHz*
25–35	2.43	0.5	0.125	29.65
35–50	1.72	0.5	0.27	41.5
150–174	0.445	0.24	0.24	157.5
400–510	0.156	0.24	0.24	435
805–865	0.0845	0.24	0.24	775

Full-scale SWR and insertion loss data are presented in figures 30–46. The difference between measured and calculated data gives a reasonable estimate of measurement uncertainty using the finite ground plane. SWR uncertainty is estimated to be the difference between the measured and calculated data. Uncertainty in monopole gain is estimated to be one-half the difference between the measured and calculated insertion loss data curves because the gain product of an identical monopole pair is an intrinsic part of the insertion loss. Measured data most closely matches the calculated data at the resonant frequencies of the monopoles, as it did for the model measurements.

The full-scale ground plane is large enough for measurements in the two highest frequency bands without resistively-loaded radials. None of the full-scale measurements show cyclical variations with frequency that would indicate reflections from the edges of the ground plane or a nearby building.

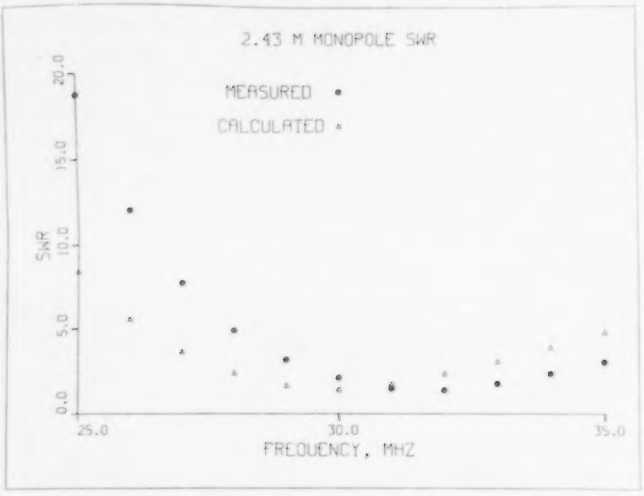


FIGURE 30. *Full-scale SWR versus frequency for the 2.43-m monopole.*

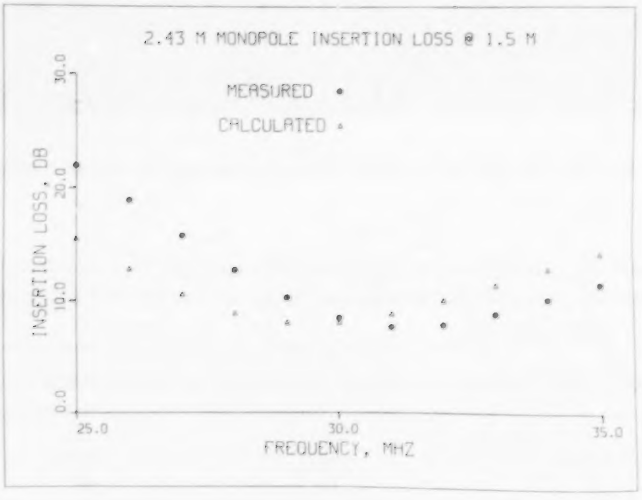


FIGURE 31. *Full-scale insertion loss versus frequency for two identical 2.43-m monopoles 1.5 m apart.*

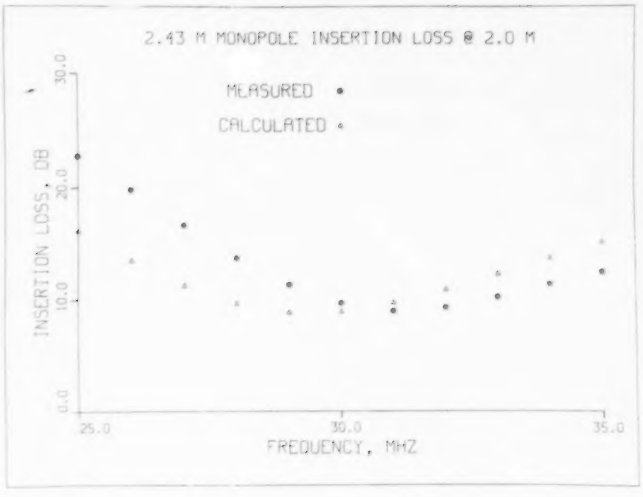


FIGURE 32. *Full-scale insertion loss versus frequency for two identical 2.43-m monopoles 2 m apart.*

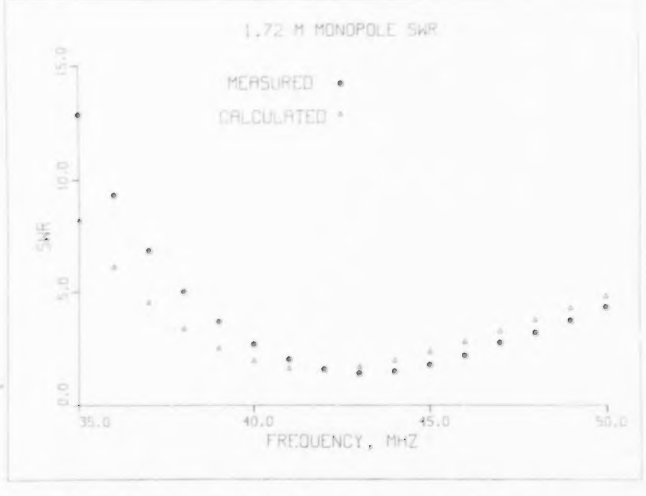


FIGURE 33. *Full-scale SWR versus frequency for the 1.72-m monopole.*

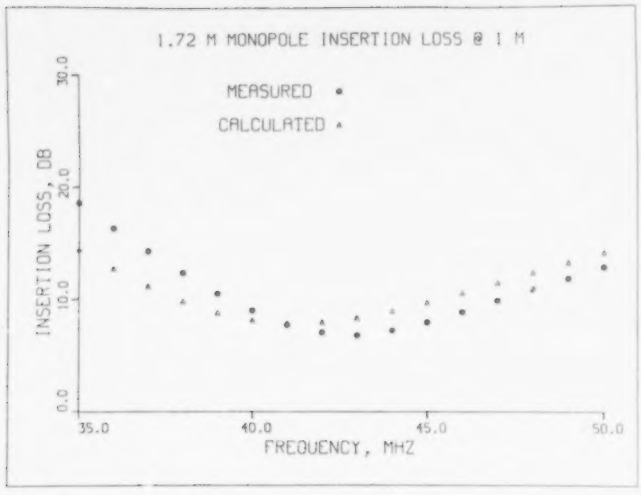


FIGURE 34. *Full-scale insertion loss versus frequency for two identical 1.72-m monopoles 1 m apart.*

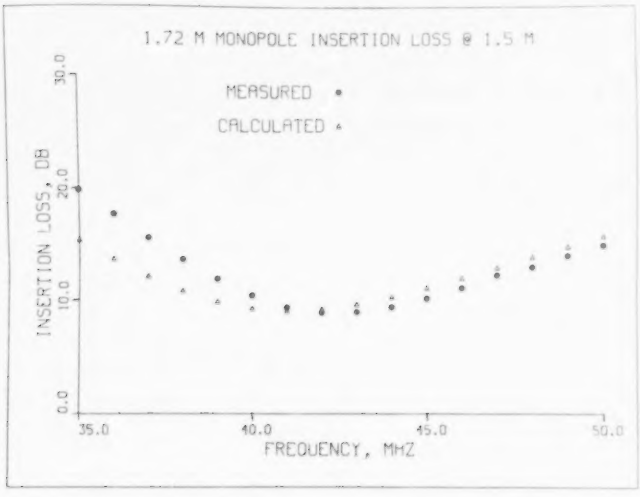


FIGURE 35. *Full-scale insertion loss versus frequency for two identical 1.72-m monopoles 1.5 m apart.*

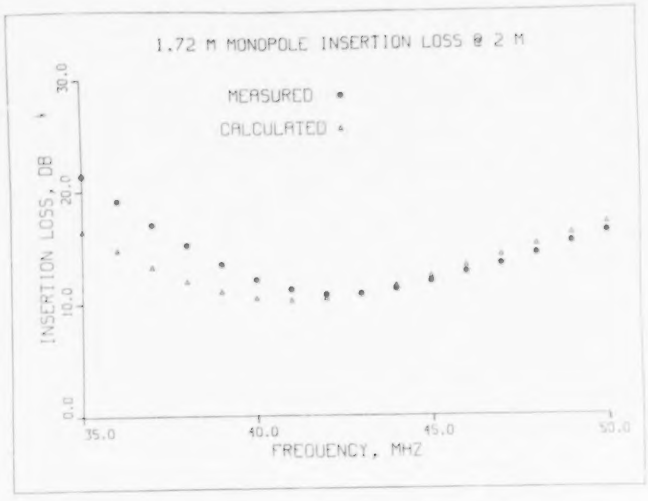


FIGURE 36. *Full-scale insertion loss versus frequency for two identical 1.72-m monopoles 2 m apart.*

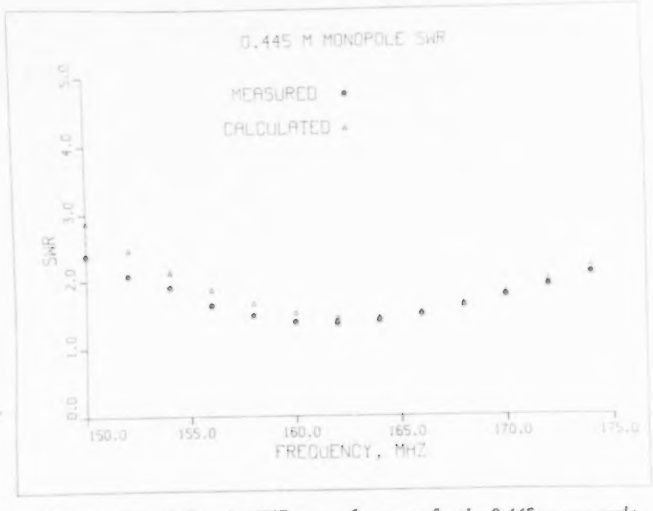


FIGURE 37. *Full-scale SWR versus frequency for the 0.445-m monopole.*

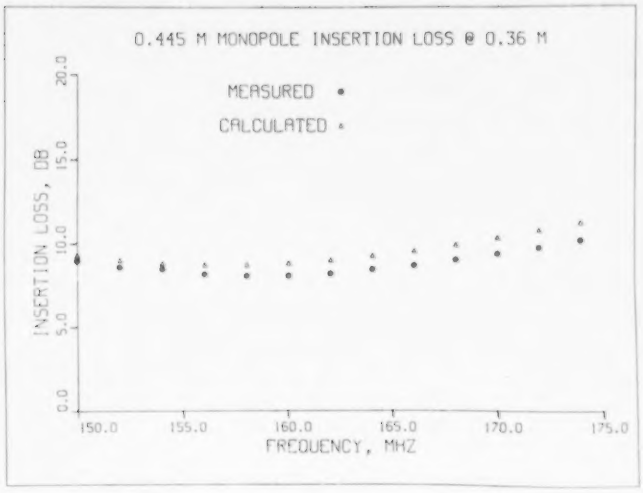


FIGURE 38. *Full-scale insertion loss versus frequency for two identical 0.445-m monopoles 0.36 m apart.*

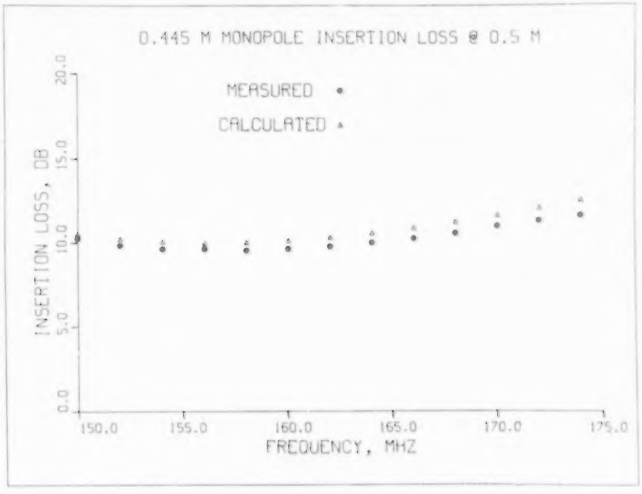


FIGURE 39. *Full-scale insertion loss versus frequency for two identical 0.445-m monopoles 0.5 m apart.*

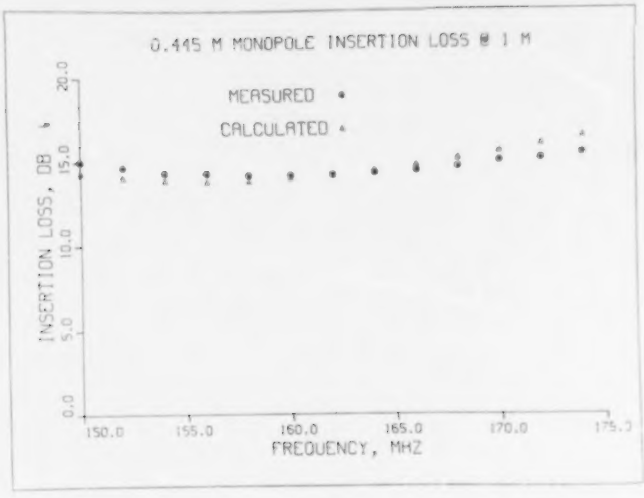


FIGURE 40. *Full-scale insertion loss versus frequency for two identical 0.445-m monopoles 1 m apart.*

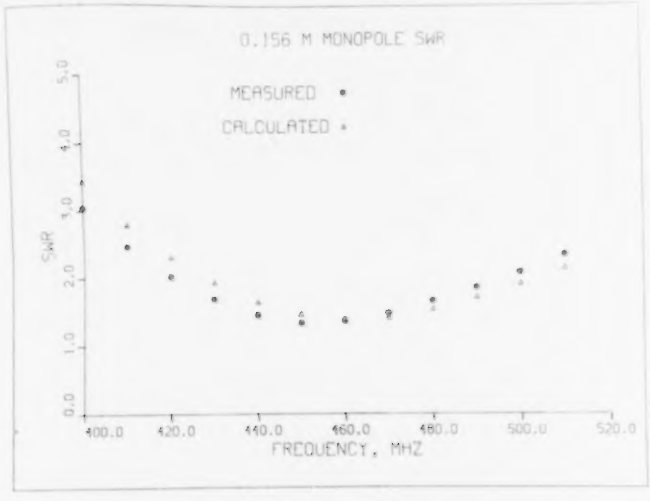


FIGURE 41. *Full-scale SWR versus frequency for the 0.156-m monopole.*

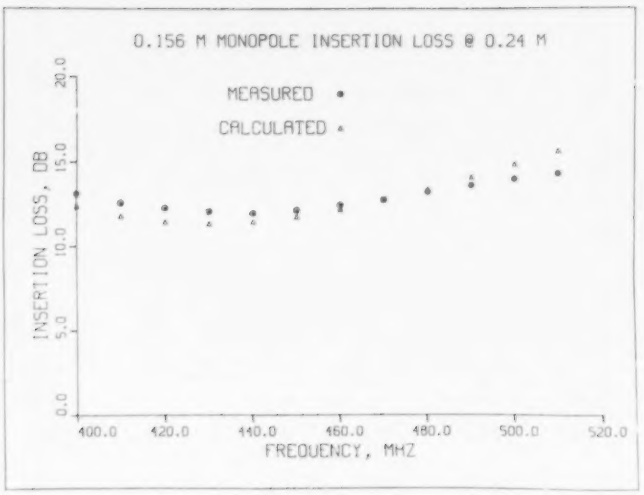


FIGURE 42. *Full-scale insertion loss versus frequency for two identical 0.156-m monopoles 0.24 m apart.*

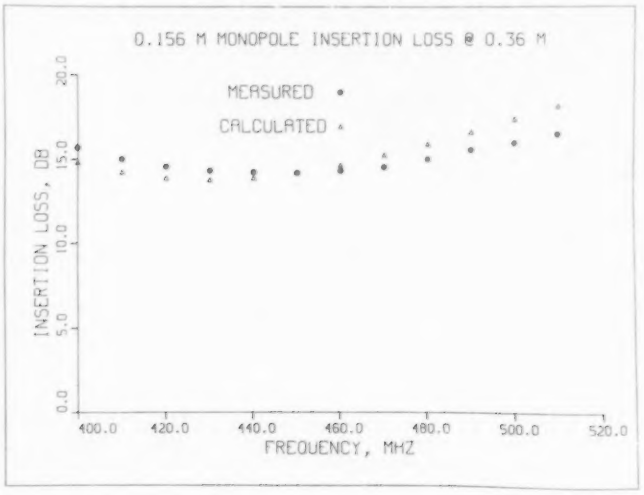


FIGURE 43. *Full-scale insertion loss versus frequency for two identical 0.156-m monopoles 0.36 m apart.*

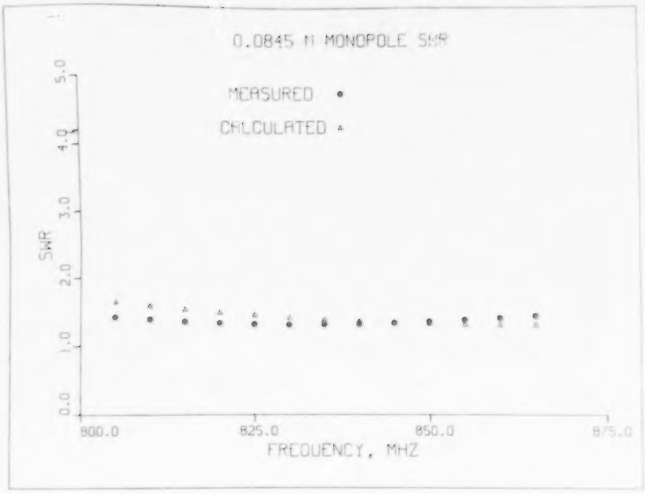


FIGURE 44. *Full-scale SWR versus frequency for the 0.0845-m monopole.*

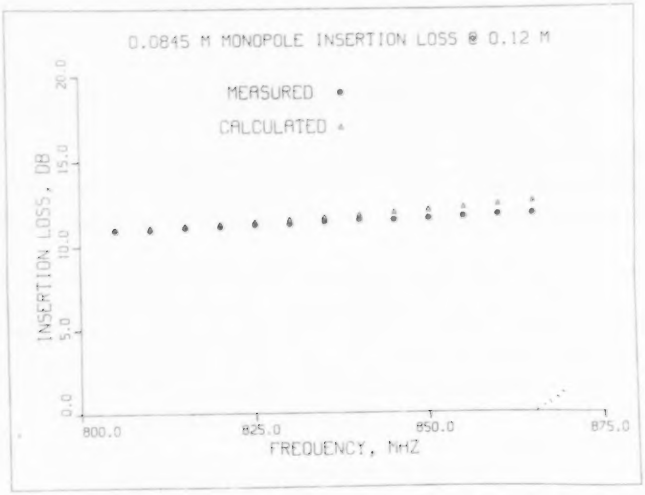


FIGURE 45. *Full-scale insertion loss versus frequency for two identical 0.0845-m monopoles 0.12 m apart.*

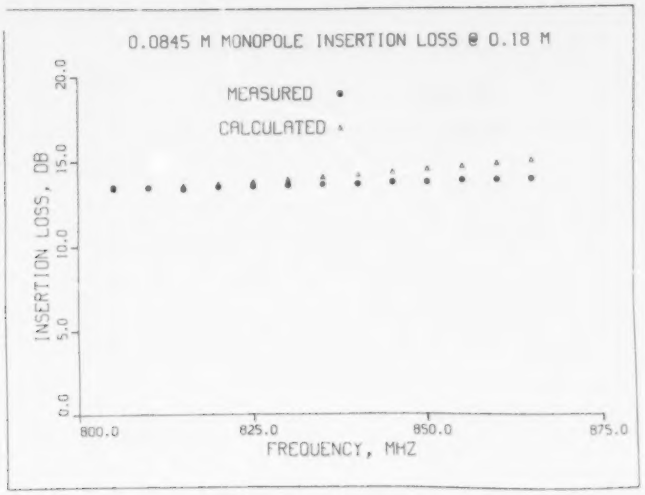


FIGURE 46. *Full-scale insertion loss versus frequency for two identical 0.0845-m monopoles 0.18 m apart.*

4. REFERENCES

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Appendix A—Listing of Program ZMIL77

```
00100 PROGRAM ZMIL34(INPUT,OUTPUT)
00110C*************************************************
00120C*  THIS PROGRAM CALCULATES INSERTION LOSS BETWEEN *
00130C*  IDENTICAL MONOPOLES ON PLANE PERFECT GROUND.   *
00135C*  IT IS A MODIFICATION OF A MORE GENERAL PROGRAM *
00136C*  AND SOME VARIABLES AND LINES OF CODE ARE       *
00137C*  REDUNDANT.                                     *
00140C*************************************************
00150 REAL IB,LP,LS,LAMBDA
00160 COMPLEX J,Z12B,Z12L,ZP,ZR,ZS
00170 10 FORMAT(F12.4)
00180 15 FORMAT(2F12.4)
00190 16 CONTINUE
00200 17 FORMAT(I5)
00210 18 FORMAT(*INPUT DATA AFTER EACH ?*)
00220 PRINT 18
00230 I=1
00240 POLAR=0
00250 30 FORMAT(*FREQUENCY, MHZ*)
00260 PRINT 30
00270 READ 10,FREQ
00280 40 FORMAT(*ANTENNA SEPARATION DISTANCE MEASURED HORIZONTALLY, M*)
00290 PRINT 40
00300 READ 10,RS
00310 45 FORMAT(*MONOPOLE LENGTH, M*)
00320 PRINT 45
00330 READ 10,LP
00340 50 FORMAT(*MONOPOLE RADIUS, M*)
00350 PRINT 50
00360 READ 10,RADIUS
00370 RRB=RADIUS
00380 RRT=RADIUS
00390 RECZ=50.
00400 HPS=0
00410 LS=LP
00420 RSB=RADIUS
00430 RST=RADIUS
00440 SSZ=50.
00450 H2=0.
00460 IF(RSB.EQ.0) RSB=1.E-30
00470 IF(RST.EQ.0) RST=1.E-30
00480 IF(RRB.EQ.0) RRB=1.E-30
00490 IF(RRT.EQ.0) RRT=1.E-30
00500 J=CMPLX(0.,1.)
00510 PI=3.14159265
00520 LAMBDA=299.792458/FREQ
00530 BETA=2.*PI/LAMBDA
```

```
00540C*****************************************************************
00550C*  ALL INPUT AND MUTUAL IMPEDANCES ARE REFERRED TO THE   *
00560C*  BASE OR TERMINAL CURRENT.                             *
00570C*****************************************************************
00580 100 FORMAT(*INPUT ZIN, R+JX*)
00590 PRINT 100
00600 READ 10,ZS
00610C*****************************************************************
00620C*  PINC IS THE INITIAL LOAD POWER FROM A SSZ-OHM GENERATOR *
00630C*****************************************************************
00640 PINC = 1.0
00650 PTR = PINC*(1.-(CABS((ZS-SSZ)/(ZS+SSZ)))**2)
00660 IB = SQRT(PTR/REAL(ZS))
00670 150 CALL ZZ(BETA,HPS,H2,J,LP,LS,LAMBDA,RS,Z12B,Z12L,POLAR)
00680 Z12B=Z12B/2.
00690 Z12L=Z12L/2.
00700 VOC=CABS(IB*Z12B)
00710 E=VOC/((LAMBDA/PI)*TAN(BETA*LP/2.))
00720 ZP=ZS
00730 PREC=((VOC/CABS(RECZ+ZP))**2)*RECZ
00740 SLOSS=10.*ALOG10(PINC/PREC)
00750C*****************************************************************
00760C*  PREC IS THE FINAL LOAD POWER MEASURED USING A RECZ-OHM*
00770C*  RECEIVER.  SLOSS IS THE INSERTION LOSS.               *
00780C*****************************************************************
00790 250 FORMAT(///)
00800 PRINT 250
00810 300 FORMAT(*FREQUENCY IS *F6.1* MHZ*)
00820 PRINT 300,FREQ
00830 550 FORMAT(*WITH AN INPUT IMPEDANCE OF*F10.2*+J[*F8.2*] OHMS*)
00840 610 FORMAT(*ANTENNAS ARE MONOPOLES*F6.2* M APART*)
00850 IF(HPS.EQ.0) PRINT 610,RS
00860 PRINT 550,ZP
00870 650 FORMAT(*MUTUAL IMPEDANCE IS*F10.3*+J[*F10.3*] OHMS*)
00880 651 FORMAT(*REFERRED TO THE BASE AND*)
00890 652 FORMAT(*REFERRED TO THE CURRENT LOOP*)
00900 PRINT 650,Z12B
00910 PRINT 651
00920 PRINT 650,Z12L
00930 PRINT 652
00940 700 FORMAT(*INSERTION LOSS IS* F8.2* DB*)
00950 PRINT 700,SLOSS
00960 PRINT 250
00970 750 FORMAT(*NEW DISTANCE OR 0 TO STOP*)
00980 PRINT 750
00990 READ 10,RS
01000 IF(RS.GT.0) GO TO 150
01010 GO TO 1000
01020 1000 CONTINUE
01030 END
```

```
01040C*************************************************************************
01050     SUBROUTINE ZZ(BETA,HPS,HS,J,LP,LS,LAMBDA,RS,Z12B,Z12L,POLAR)
01060C*  COMPUTES THE MUTUAL IMPEDANCE BETWEEN IDENTICAL MONOPOLES.      *
01100C*************************************************************************
01110 DIMENSION ARG(18),SI(18),CI(18),CIN(18),SIS(18),
01120+          SIL(18),CIS(18),CIL(18)
01130 REAL L,LP,LS,LAMBDA
01140 COMPLEX J,Z12L,Z12B
01150 B=BETA
01160 M=0
01170 10 HP=HPS
01180 IF(POLAR.EQ.0.) D=RS
01190 IF(POLAR.EQ.1.AND.M.EQ.0.) D=SQRT(RS*RS+(HS-HP)**2)
01200 IF(POLAR.EQ.1.AND.M.EQ.1.) D=SQRT(RS*RS+(HS+HP)**2)
01210 IF(POLAR.EQ.0.AND.M.EQ.0.) H=HS-HP-LS
01220 IF(POLAR.EQ.1.) H=-LS
01230 IF(POLAR.EQ.0.AND.M.EQ.1.) H=HS+HP-LS
01240 ARG(1)=B*(SQRT(D*D+(H-LP)**2)+(H-LP))
01250 ARG(2)=B*(SQRT(D*D+(H-LP)**2)-(H-LP))
01260 ARG(3)=B*(SQRT(D*D+(H+LP)**2)-(H+LP))
01270 ARG(4)=B*(SQRT(D*D+(H+LP)**2)+(H+LP))
01280 ARG(5)=B*(SQRT(D*D+(H-LP+LS)**2)+(H-LP+LS))
01290 ARG(6)=B*(SQRT(D*D+(H-LP+LS)**2)-(H-LP+LS))
01300 ARG(7)=B*(SQRT(D*D+(H+LP+LS)**2)-(H+LP+LS))
01310 ARG(8)=B*(SQRT(D*D+(H+LP+LS)**2)+(H+LP+LS))
01320 ARG(9)=B*(SQRT(D*D+(H-LP+2.*LS)**2)+(H-LP+2.*LS))
01330 ARG(10)=B*(SQRT(D*D+(H-LP+2.*LS)**2)-(H-LP+2.*LS))
01340 ARG(11)=B*(SQRT(D*D+(H+LP+2.*LS)**2)-(H+LP+2.*LS))
01350 ARG(12)=B*(SQRT(D*D+(H+LP+2.*LS)**2)+(H+LP+2.*LS))
01360 ARG(13)=B*(SQRT(D*D+H*H)-H)
01370 ARG(14)=B*(SQRT(D*D+H*H)+H)
01380 ARG(15)=B*(SQRT(D*D+(H+LS)**2)-(H+LS))
01390 ARG(16)=B*(SQRT(D*D+(H+LS)**2)+(H+LS))
01400 ARG(17)=B*(SQRT(D*D+(H+2.*LS)**2)-(H+2.*LS))
01410 ARG(18)=B*(SQRT(D*D+(H+2.*LS)**2)+(H+2.*LS))
01420 DO 100 I=1,18
01430 100 CALL SCI(ARG(I),SI(I),CI(I),CIN(I))
01440C*************************************************************
01450C*  COMPUTE MUTUAL IMPEDANCE USING EQUATIONS FROM H.E. KING  *
01460C*************************************************************
01470 R12=15.*(COS(B*(LP-H))*(CI(1)+CI(2)-CI(5)-CI(6))+SIN(B*(LP-H))
01480+*(-SI(1)+SI(2)+SI(5)-SI(6))+COS(B*(LP+H))*(CI(3)+CI(4)-CI(7)-CI(8))
01490++SIN(B*(LP+H))*(-SI(3)+SI(4)+SI(7)-SI(8))+COS(B*(LP-2.*LS-H))*
01500+(-CI(5)-CI(6)+CI(9)+CI(10))+SIN(B*(LP-2.*LS-H))*(SI(5)-SI(6)-SI(9)
01510++SI(10))+COS(B*(LP+2.*LS+H))*(-CI(7)-CI(8)+CI(11)+CI(12))+SIN(B*
01520+(LP+2.*LS+H))*(SI(7)-SI(8)-SI(11)+SI(12))+2.*COS(B*LP)*COS(B*H)*
01530+(-CI(13)-CI(14)+CI(15)+CI(16))+2.*COS(B*LP)*SIN(B*H)*(SI(13)-SI(14)
01540+-SI(15)+SI(15))+2.*COS(B*LP)*COS(B*(2.*LS+H))*(CI(15)+CI(16)-CI(17)
01550+-CI(18))+2.*COS(B*LP)*SIN(B*(2.*LS+H))*(-SI(15)+SI(16)+SI(17)
01560+-SI(18)))
01570 X12=15.*(COS(B*(LP-H))*(-SI(1)-SI(2)+SI(5)+SI(6))+SIN(B*(LP-H))
01580+*(-CI(1)+CI(2)+CI(5)-CI(6))+COS(B*(LP+H))*(-SI(3)-SI(4)+SI(7)+SI(8))
```

```
01590++SIN(B*(LP+H))*(-CI(3)+CI(4)+CI(7)-CI(8))+COS(B*(LP-2.*LS-H))*
01600+(+SI(5)+SI(6)-SI(9)-SI(10))+SIN(B*(LP-2.*LS-H))*(CI(5)-CI(6)-CI(9)
01610++CI(10))+COS(B*(LP+2.*LS+H))*(+SI(7)+SI(8)-SI(11)-SI(12))+SIN(B*
01620+(LP+2.*LS+H))*(CI(7)-CI(8)-CI(11)+CI(12))+2.*COS(B*LP)*COS(B*H)*
01630+(+SI(13)+SI(14)-SI(15)-SI(16))+2.*COS(B*LP)*SIN(B*H)*(CI(13)-CI(14)
01640+-CI(15)+CI(16))+2.*COS(B*LP)*COS(B*(2.*LS+H))*(-SI(15)-SI(16)+SI(17)
01650++SI(18))+2.*COS(B*LP)*SIN(B*(2.*LS+H))*(-CI(15)+CI(16)+CI(17)
01660+-CI(18)))
01670  Z12L=R12+J*X12
01680 Z12B=Z12L/((SIN(BETA*LP)*SIN(BETA*LS)))
01690C*************************************************************************
01700C*  THE TERMS IN THE PRECEEDING LINE FOLLOWING THE / CHANGE THE  *
01710C*  MUTUAL IMPEDANCE REFERENCE FROM THE LOOP CURRENT TO THE BASE *
01720C*  CURRENT.  SEE JORDAN PP. 345-351.                            *
01730C*************************************************************************
01740 RETURN
01750 END
01760C*************************************************************************
01770      SUBROUTINE SCI(ARG,SI,CI,CIN)
01780C*   COMPUTES THE SINE, COSINE, AND MODIFIED COSINE INTEGRALS  *
01790C*************************************************************************
01800 DIMENSION T(44),S(21),C(21)
01810 K=1
01820 T(1)=1.
01830 DO 10 M=1,42
01840 K=K+1
01850 10 T(K)=M*T(K-1)
01860 Z=ARG
01870 IF(Z.GT.2.) GO TO 40
01880C*********************************************
01890C*  COMPUTE THE INTEGRALS FOR 0<ARG(I)<1  *
01900C*********************************************
01910 DO 20 N=1,20
01920 20 S(N)=((-1)**N)*(Z**(2*N+1))/((2*N+1)*T(2*N+2))
01930 SIS=Z+S(1)+S(2)+S(3)+S(4)+S(5)+S(6)+S(7)+S(8)+S(9)+S(10)+S(11)
01940+        +S(12)+S(13)+S(14)+S(15)+S(16)+S(17)+S(18)+S(19)+S(20)
01950 DO 30 N=1,20
01960 30 C(N)=((-1)**N)*(Z**(2*N))/((2*N)*T(2*N+1))
01970 SA = C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)+C(9)+C(10)+C(11)
01980+      +C(12)+C(13)+C(14)+C(15)+C(16)+C(17)+C(18)+C(19)+C(20)
01990 CIS = 0.577215665+ALOG(Z)+SA
02000 CINS=-SA
02010 IF(Z.LE.1.) GO TO 50
02020C*****************************************************
02030C*  COMPUTE THE INTEGRALS FOR 1<ARG(I)<INFINITY *
02040C*****************************************************
02050 40 FZ=(Z**8+38.027264*(Z**6)+265.187033*(Z**4)+335.67732*(Z**2)
02060+       +38.102495)/(Z*((Z**8)+40.021433*(Z**6)+322.624911*(Z**4)
02070+       +570.23628*(Z**2)+157.105423))
02080 GZ=(Z**8+42.242855*(Z**6)+302.757865*(Z**4)+352.018498*(Z**2)
02090+    +21.821899)/((Z**2)*((Z**8)+48.196927*(Z**6)+482.485984
02100+    *(Z**4)+1114.978885*(Z**2)+449.690326))
```

```
02110 SIL=1.570796327-FZ*COS(Z)-GZ*SIN(Z)
02120 CIL=FZ*SIN(Z)-GZ*COS(Z)
02130 CINL=-CIL+0.5772157+ALOG(Z)
02140C*****************************************************************
02150C*  THE FOLLOWING SIX LINES ARE AN INTERPOLATION ROUTINE TO  *
02160C*  SMOOTH THE TRANSITION REGION BETWEEN THE LARGE AND SMALL *
02170C*  ARGUMENT APPROXIMATIONS TO THE SINE AND COSINE INTEGRALS.*
02180C*****************************************************************
02190 50 IF(Z.LE.1.) SI=SIS
02200 IF(Z.LE.1.) CI=CIS
02210 IF(Z.LE.1.) CIN=CINS
02220 IF(Z.GT.1.AND.Z.LE.2.) SI=SIS+(Z-1.)*(SIL-SIS)
02230 IF(Z.GT.1.AND.Z.LE.2.) CI=CIS+(Z-1.)*(CIL-CIS)
02240 IF(Z.GT.1.AND.Z.LE.2.) CIN=CINS+(Z-1.)*(CINL-CINS)
02250 IF(Z.GT.2.) SI=SIL
02260 IF(Z.GT.2.) CI=CIL
02270 IF(Z.GT.2.) CIN=CINL
02280 RETURN
02290 END
```

Appendix B—Listing of Program GAIN77

```
00100 PROGRAM HVD6(INPUT,OUTPUT,TAPE50)
00110C*************************************************************************
00120C*  GAIN VERSUS ELEVATION ANGLE AND IMPEDANCE RELATED PARAMETERS  *
00130C*  FOR VERTICAL AND HORIZONTAL DIPOLES OVER GROUND AND MONOPOLES *
00140C*  ON PERFECT GROUND.............R.G. FITZGERRELL (303) 497-3737 *
00150C*  TAPE50 IS USED IN A PLOTTING PROCEDURE FILE.  DELETE THE FOUR *
00160C*  CALLS TO FILE AND SUBROUTINE FILE IF DESIRED.                  *
00170C*************************************************************************
00180 DIMENSION ARG(2),CI(2),CIN(2),SI(2)
00190 COMPLEX E,J,JWCT,R,RH,RV,RH90,RV90,TSQ,ZA,ZIND,ZINN,ZML,ZR,ZI,ZMB
00200 REAL KA,LAMBDA,L,L2,LOSS,NBL,MBL,MONO
00210C*************************************************************************
00220C*  USER SPECIFIES ANTENNA GEOMETRY.  ANTENNA ELEMENTS MAY *
00230C*  HAVE A LINEAR TAPER OR A ZERO (1.E-30) RADIUS.         *
00240C*************************************************************************
00250 10 FORMAT(*TYPE THE REQUESTED DATA AFTER EACH ?*)
00260 PRINT 10
00270 20 FORMAT(*FREQUENCY, MHZ*)
00280 PRINT 20
00290 30 FORMAT(F12.4)
00300 READ 30,FREQ
00310 40 FORMAT(*ANTENNA HALF-LENGTH < LAMBDA/2, M*)
00320 PRINT 40
00330 READ 30,L
00340 50 FORMAT(*ANTENNA RADII, FEEDPOINT THEN END, M*)
00350 PRINT 50
00360 READ 30,RB
00370 READ 30,RT
00380 IF(RT.EQ.0.)RT=1.E-30
00390 IF(RB.EQ.0.)RB=1.E-30
00400 60 FORMAT(*TYPE 1. FOR DIPOLE, 0 FOR MONOPOLE*)
00410 PRINT 60
00420 READ 30, MONO
00430 IF(MONO.EQ.0)POLAR=0.
00440 IF(MONO.EQ.0) GO TO 140
00450 70 FORMAT(*TYPE 1. FOR GROUND, 0 FOR FREE SPACE*)
00460 PRINT 70
00470 READ 30,GF
00480 IF(GF.EQ.0.)POLAR=1.
00490 IF(GF.EQ.0) GO TO 140
00500 80 FORMAT(*DIPOLE FEED-POINT HEIGHT ABOVE GROUND, M*)
00510 PRINT 80
00520 READ 30, H
00530 IF(H.LE.L)POLAR=1.
00540 IF(H.LE.L) GO TO 95
00550 90 FORMAT(*TYPE 1. FOR HORIZONTAL POLARIZATION, 0 FOR VERTICAL*)
00560 PRINT 90
00570 READ 30, POLAR
00580 95 CONTINUE
```

```
00590 100 FORMAT(*TYPE 1. FOR PERFECT GROUND, 0 FOR IMPERFECT GROUND*)
00600 PRINT 100
00610 READ 30, GND
00620 IF(GND.EQ.1.) GO TO 130
00630 110 FORMAT(*GROUND CONDUCTIVITY, MILLIMHOS/M*)
00640 PRINT 110
00650 READ 30,SIGMA
00660 120 FORMAT(*GROUND RELATIVE DIELECTRIC CONSTANT*)
00670 PRINT 120
00680 READ 30,EPSN
00690 GO TO 150
00700C*****************************************************************
00710C*  PERFECT GROUND AND FREE-SPACE APPROXIMATIONS ARE MADE. *
00720C*****************************************************************
00730 130 SIGMA=1.E30
00740 GO TO 145
00750 140 SIGMA=0.0
00760 145 EPSN=1.0
00770 150 CONTINUE
00780 160 FORMAT(*IMPEDANCE OF T-LINE ATTACHED TO ANTENNA, OHMS*)
00790 PRINT 160
00800 READ 30,ZCHAR
00810 J=CMPLX(0.,1.)
00820 LAMBDA=299.792458/FREQ
00830 BETA=6.2831853/LAMBDA
00840 X=17.975*SIGMA/FREQ
00850 170 FORMAT(*- - - - - - - - - - - - - - - - - - - - - - - - - - - - -*)
00860 PRINT 170
00870C*********************************
00880C*  THE CALCULATIONS BEGIN HERE  *
00890C*********************************
00900 200 B=2.*BETA*L
00910 BL=BETA*L
00920 DO 210 M=1,2
00930 ARG(M)=M*B
00940 210 CALL SCI(ARG(M),SI(M),CI(M),CIN(M))
00950C*****************************************************************
00960C*  COMPUTE SELF-IMPEDANCE, ZI, USING SCHELKUNOFF'S EQN. 13-108  *
00970C*****************************************************************
00980 IF(RT.EQ.RB)KA=120.*(ALOG(2.*L/RT)-1.)
00990 IF(RT.NE.RB)KA=120.*(ALOG(2*L/RB)+(RT/(RB-RT))*ALOG(RT/RB))
01000 MBL=60.*(CIN(1)-1.+COS(B))
01010 NBL=60.*(SI(1)-SIN(B))
01020 RABL=60.*CIN(1)+30.*(2.*CIN(1)-CIN(2))*COS(B)+30.*(SI(2)
01030+       -2.*SI(1))*SIN(B)
01040 XABL=60.*SI(1)-30.*(CIN(2)-ALOG(4.))*SIN(B)-30.*SI(2)*COS(B)
01050 ZA=RABL+J*XABL
01060 JWCT=J*RT/(30.*LAMBDA)
01070 ZINN=KA*((KA-MBL)*COS(BL)+J*(ZA+JWCT*KA*KA-J*NBL)*SIN(BL))
01080 ZIND=(ZA+JWCT*KA*KA+J*NBL)*COS(BL)+J*(KA+MBL)*SIN(BL)
01090 ZI=ZINN/ZIND
01100C*****************************************************************
```

```
01110C*  COMPUTE ANTENNA INPUT IMPEDANCE, ZR.  SUBROUTINE MUTUAL RETURNS *
01120C*  MUTUAL IMPEDANCE REFERRED TO THE LOOP CURRENT, ZM, BETWEEN TWO  *
01130C*  FREE-SPACE DIPOLES. SIN(BL)**2 CHANGES REFERENCE TO BASE CURRENT*
01140C*************************************************************************
01150 IF(SIGMA.EQ.0.AND.MONO.EQ.1.)ZR=ZI
01160 IF(MONO.EQ.0.)ZR=ZI/2.
01170 IF(MONO.EQ.0.OR.SIGMA.EQ.0.) GO TO 240
01180 CALL MUTUAL(BETA,H,J,L,ZML,POLAR)
01190 ZMB=ZML/(SIN(BL)**2)
01200 IF(H.GE.0.1*L) GO TO 230
01210 H1=0.0001*LAMBDA
01220 POLAR1=1.0
01230 CALL MUTUAL(BETA,H1,J,L,ZML,POLAR1)
01240 ZI=ZML/(SIN(BL)**2)
01250C*****************************************************************
01260C*  THE ABOVE 5 LINES HELP PREVENT NEGATIVE VALUES OF ZR FOR  *
01270C*  DIPOLES NEAR THE GROUND BECAUSE SCHELKUNOFF'S ZI DOES NOT *
01280C*  CONVERGE TO THAT OBTAINED USING SUBROUTINE MUTUAL.        *
01290C*****************************************************************
01300C*  COMPUTE REFLECTION COEFFICIENTS FOR PSI=90 DEG  *
01310C*****************************************************
01320 230 RV90=((EPSN-J*X)-CSQRT(EPSN-J*X))/((EPSN-J*X)+CSQRT(EPSN-J*X))
01330 RH90=(1.-CSQRT(EPSN-J*X))/(1.+CSQRT(EPSN-J*X))
01340 IF(POLAR.EQ.1.AND.MONO.EQ.1.)ZR=ZI+RH90*ZMB
01350 IF(POLAR.EQ.0.AND.MONO.EQ.1.)ZR=ZI+RV90*ZMB
01360 240 CONTINUE
01370 250 FORMAT(*INPUT IMPEDANCE =*F12.4*+J[*F12.4*]OHMS*)
01380 PRINT 250,ZR
01390 PRINT 170
01400 260 FORMAT(*ENTER NEW HALF-LENGTH, OR ENTER 0*)
01410 PRINT 260
01420 READ 30,L2
01430 IF(L2.GT.0) L=L2
01440 IF(L2.GT.0) GO TO 200
01450C*************************************************************************
01460C*  COMPUTE ANTENNA FACTOR; MUTUAL IMPEDANCE; VSWR; MISMATCH LOSS*
01470C*  ANTENNA FACTOR=VOLTAGE DIVIDER FACTOR/EFFECTIVE LENGTH       *
01480C*************************************************************************
01490 IF(MONO.EQ.0.)AF=(CABS(ZCHAR+ZR)/ZCHAR)/ABS((1./BETA)*TAN(BL/2.))
01500 IF(MONO.EQ.1.)AF=(CABS(ZCHAR+ZR)/ZCHAR)/ABS((2./BETA)*TAN(BL/2.))
01510 PRINT 170
01520 300 FORMAT(*ANTENNA FACTOR ASSUMING A *F6.2*-OHM RECEIVER*)
01530 310 FORMAT(*AND A 1:1 BALUN ARE EMPLOYED =*F8.2* DB*)
01540 320 FORMAT(*AND TRANSMISSION LINE ARE EMPLOYED =*F8.2* DB*)
01550 PRINT 300,ZCHAR
01560 IF(MONO.EQ.1.) PRINT 310, 20.*ALOG10(AF)
01570 IF(MONO.EQ.0.) PRINT 320, 20.*ALOG10(AF)
01580 330 FORMAT(*MUTUAL IMPEDANCE DUE TO THE DIPOLE'S IMAGE*/
01590+*IN THE GROUND =*F12.4*+J[*F12.4*]OHMS*)
01600 IF(SIGMA.GT.0.) PRINT 330, ZR-ZI
01610 R=(ZR-ZCHAR)/(ZR+ZCHAR)
01620 VSWR=(1.+CABS(R))/(1.-CABS(R))
```

```
01630 340 FORMAT(*VSWR =*F12.4)
01640 PRINT 340,VSWR
01650 LOSS=10.0*ALOG10(1.0/(1.0-CABS(R*R)))
01660 350 FORMAT(*MISMATCH LOSS =*F12.4* DB*)
01670 PRINT 350,LOSS
01680 PRINT 170
01690C*********************************************************
01700C*  COMPUTE GAIN VERSUS ELEVATION OR EXIT PROGRAM.  *
01710C*********************************************************
01720 400 FORMAT(*PRESS RETURN FOR GAIN VS PHI, OTHERWISE TYPE CNTL T*)
01730 PRINT 400
01740 PAUSE
01750 410 FORMAT(*ELEV ANGLE INCREMENT FOR BELOW 10 DEG AND*/
01760+*THE ADDITIONAL INCREMENT FOR ABOVE 10 DEG*)
01770 PRINT 410
01780 READ 30, ENC
01790 READ 30, ENC2
01800 CALL FILE(ENC,ENC2,L,POLAR,SIGMA,0.,0.)
01810 IF(SIGMA.EQ.0.) H=1.
01820 BH=BETA*H
01830 IF(POLAR.EQ.0) GO TO 530
01840 PRINT 170
01850 420 FORMAT(*H-PLANE ELEVATION ANGLE IN DEGREES AND GAIN IN DB*)
01860 PRINT 420
01870 430 FORMAT(*HORIZONTAL POLARIZATION*)
01880 PRINT 430
01890 PSI=0
01900C*****************************
01910C*  HORIZONTAL POLARIZATION  *
01920C*****************************
01930 500  PSI=PSI+ENC
01940 IF (PSI.LT.10.1) GO TO 510
01950 PSI=PSI+ENC2
01960 IF(PSI.GT.90.) GO TO 530
01970 510 PSIRAD=0.01745329252*PSI
01980 SNPSI=SIN(PSIRAD)
01990 CNPSI=COS(PSIRAD)
02000 TSQ=CSQRT(EPSN-J*X-(CNPSI**2))
02010 RH=(SNPSI-TSQ)/(SNPSI+TSQ)
02020C*************************************************************
02030C*  RH & RV ARE HORIZONTAL AND VERTICAL REFLECTION COEFFICIENTS *
02040C*************************************************************
02050 EFLH=ABS(TAN(BL/2.))
02060 E=EFLH*(CEXP(J*BH*SNPSI)+RH*(CEXP(-J*BH*SNPSI)))
02070 E2=120.*CABS(E*E)/REAL(ZR)
02080C*************************************************************
02090C*  THE GAIN EQUATION IS BASICALLY JORDAN'S EQN. 12-18 USING     *
02100C*  (EFFECTIVE LENGTH)*PI/LAMBDA DENOTED AS E AND APPROPRIATELY  *
02110C*  DEFINED FOR THE ORIENTATION OF THE DIPOLE.                   *
02120C*************************************************************
02130 GAIN=10.0*ALOG10(E2)
02140 520 FORMAT(2F15.3)
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```
02150 PRINT 520,PSI,GAIN
02160 CALL FILE(-1.,-1.,-1.,-1.,-1.,PSI,GAIN)
02170 GO TO 500
02180 530 PRINT 170
02190 IF(POLAR.GT.0) PAUSE
02200 540 FORMAT(*E-PLANE ELEVATION ANGLE IN DEGREES AND GAIN IN DB*)
02210 545 FORMAT(*VERTICAL POLARIZATION*)
02220 PRINT 540
02230 PRINT 545
02240 PSI=0
02250C*****************************
02260C*  VERTICAL POLARIZATION  *
02270C*****************************
02280 550 PSI=PSI+ENC
02290 IF (PSI.LT.10.1) GO TO 560
02300 PSI=PSI+ENC2
02310 IF (PSI.GT.90.) GO TO 580
02320 560 PSIRAD=0.01745329252*PSI
02330 SNPSI=SIN(PSIRAD)
02340 CNPSI=COS(PSIRAD)
02350 TSQ=CSQRT(EPSN-J*X-(CNPSI**2))
02360 RV=(((EPSN-J*X)*SNPSI)-TSQ)/(((EPSN-J*X)*SNPSI)+TSQ)
02370 IF(POLAR.EQ.1.) GO TO 570
02380 EFLV=ABS(COS(BL*SNPSI)-COS(BL))/(CNPSI*SIN(BL))
02390 E=EFLV*(CEXP(J*BH*SNPSI)+RV*(CEXP(-J*BH*SNPSI)))
02400 E2=120.*CABS(E*E)/REAL(ZR)
02410 IF(E2.EQ.0.) E2=1.E-12
02420 GAIN=10.0*ALOG10(E2)
02430 PRINT 520,PSI,GAIN
02440 CALL FILE(-1.,-1.,-1.,-1.,-1.,PSI,GAIN)
02450 GO TO 550
02460C*****************************************************
02470C*  VERTICAL POLARIZATION FOR A HORIZONTAL DIPOLE  *
02480C*****************************************************
02490 570 EFLVH=ABS(COS(BL*CNPSI)-COS(BL))/(SNPSI*SIN(BL))
02500 E=EFLVH*(CEXP(J*BH*SNPSI)-RV*(CEXP(-J*BH*SNPSI)))
02510 E2=120.0*CABS(E*E)/REAL(ZR)
02520 GAIN=10.0*ALOG10(E2)
02530 PRINT 520,PSI,GAIN
02540 CALL FILE(-1.,-1.,-1.,-1.,-1.,PSI,GAIN)
02550 GO TO 550
02560 580 PRINT 170
02570 END
02580C*****************************************************************
02590      SUBROUTINE MUTUAL(BETA,H,J,L,ZML,POLAR)
02600C*    COMPUTES MUTUAL IMPEDANCES BETWEEN DIPOLES IN ECHELON *
02610C*****************************************************************
02620 DIMENSION ARG(5),SI(5),CI(5),CIN(5)
02630 REAL L,L1,L2,L3,L4
02640 COMPLEX J,ZML
02650C*********************************************************
02660C*  COMPUTE ARGUMENTS FOR SINE AND COSINE INTEGRALS  *
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02670C*************************************************************
02680 Y0=0.
02690 X0=2.*H
02700 IF(POLAR.EQ.0.) GO TO 1
02710 Y0=2.*H
02720 X0=0.
02730C*************************************************************************
02740C*  X0 IS THE SEPARATION BETWEEN THE FEEDPOINTS OF THE VERTICAL  *
02750C*  DIPOLE AND ITS COLINEAR IMAGE.  Y0 IS THE SEPARATION BETWEEN *
02760C*  THE HORIZONTAL DIPOLE AND ITS IMAGE.                         *
02770C*************************************************************************
02780 1 B=BETA
02790 Y02=Y0**2
02800 HL1=X0-2.*L
02810 HL2=X0-L
02820 HL3=X0
02830 HL4=X0+L
02840 HL5=X0+2.*L
02850 ARG(1)=B*(SQRT(Y02+HL1**2)+HL1)
02860 ARG(2)=B*(SQRT(Y02+HL2**2)+HL2)
02870 ARG(3)=B*(SQRT(Y02+HL3**2)+HL3)
02880 ARG(4)=B*(SQRT(Y02+HL4**2)+HL4)
02890 ARG(5)=B*(SQRT(Y02+HL5**2)+HL5)
02900 DO 10 I=1,5
02910 10 CALL SCI(ARG(I),SI(I),CI(I),CIN(I))
02920 IF(POLAR.EQ.0.) GO TO 20
02930C*************************************************************
02940C*  COMPUTE MUTUAL IMPEDANCE FOR HORIZONTAL POLARIZATION  *
02950C*************************************************************
02960 R12=30.*(COS(2.*B*L)*(CI(1)+CI(5)-2.*CI(2)-2.*CI(4)+2.*CI(3))
02970+         +SIN(2.*B*L)*(-SI(1)+SI(5)+2.*SI(2)-2.*SI(4))
02980+         +4.*CI(3)-2.*CI(2)-2.*CI(4))
02990 X12=30.*(COS(2.*B*L)*(-SI(1)-SI(5)+2.*SI(2)+2.*SI(4)-2.*SI(3))
03000+         +SIN(2.*B*L)*(-CI(1)+CI(5)+2.*CI(2)-2.*CI(4))
03010+         -4.*SI(3)+2.*SI(2)+2.*SI(4))
03020 ZML=R12+J*X12
03030 RETURN
03040C*************************************************************
03050C*  COMPUTE MUTUAL IMPEDANCE FOR VERTICAL POLARIZATION  *
03060C*************************************************************
03070 20 L1=ALOG(HL2/HL1)
03080 L2=ALOG(HL4/HL3)
03090 L3=ALOG(HL2/HL3)
03100 L4=ALOG(HL4/HL5)
03110 B1=B*HL1
03120 B2=B*HL3
03130 B3=B*HL5
03140 B4=B*HL2
03150 B5=B*HL4
03160 R12=15.*(COS(B1)*(CI(1)-CI(2)+L1)+SIN(B1)*(SI(1)-SI(2))+COS(B2)
03170+     *(CI(3)-CI(4)+L2)+SIN(B2)*(SI(3)-SI(4))+COS(B2)*(-CI(2)+CI(3)
03180+     +L3)+SIN(B2)*(-SI(2)+SI(3))+COS(B3)*(-CI(4)+CI(5)+L4)+SIN(B3)
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03190+     *(-SI(4)+SI(5))+2.*COS(B*L)*COS(B4)*(-CI(2)+CI(3)+L3)+2.
03200+     *COS(B*L)*SIN(B4)*(-SI(2)+SI(3))+2.*COS(B*L)*COS(B5)*(CI(3)
03210+     -CI(4)+L2)+2.*COS(B*L)*SIN(B5)*(SI(3)-SI(4)))
03220 X12=15.*(COS(B1)*(-SI(1)+SI(2))+SIN(B1)*(CI(1)-CI(2)-L1)+COS(B2)
03230+     *(-SI(3)+SI(4))+SIN(B2)*(CI(3)-CI(4)-L2)+COS(B2)*(SI(2)
03240+     -SI(3))+SIN(B2)*(-CI(2)+CI(3)-L3)+COS(B3)*(SI(4)-SI(5))
03250+     +SIN(B3)*(-CI(4)+CI(5)-L4)+2.*COS(B*L)*COS(B4)*(SI(2)
03260+     -SI(3))+2.*COS(B*L)*SIN(B4)*(-CI(2)+CI(3)-L3)+2.*COS(B*L)
03270+     *COS(B5)*(-SI(3)+SI(4))+2.*COS(B*L)*SIN(B5)*(CI(3)-CI(4)-L2))
03280 ZML=R12+J*X12
03290 RETURN
03300 END
03310C*****************************************************************
03320       SUBROUTINE SCI(ARG,SI,CI,CIN)
03330C*     COMPUTES THE SINE, COSINE, AND MODIFIED COSINE INTEGRALS *
03340C*****************************************************************
03350 DIMENSION T(42),S(20),C(20)
03360 K=1
03370 T(1)=1.0
03380 DO 10 M=1,42
03390 K=K+1
03400 10 T(K)=M*T(K-1)
03410 IF(ARG.GT.1.) GO TO 25
03420C*****************************************
03430C*  COMPUTE THE INTEGRALS FOR 0<ARG<1   *
03440C*****************************************
03450 DO 20 N=1,20
03460 20 S(N)=((-1)**N)*(ARG**(2*N+1))/((2*N+1)*T(2*N+2))
03470 SI=ARG+S(1)+S(2)+S(3)+S(4)+S(5)+S(6)+S(7)+S(8)+S(9)+S(10)
03480+       +S(11)+S(12)+S(13)+S(14)+S(15)+S(16)+S(17)+S(18)+S(19)+S(20)
03490 DO 30 N=1,20
03500 30 C(N)=((-1)**N)*(ARG**(2*N))/((2*N)*T(2*N+1))
03510 SA=C(1)+C(2)+C(3)+C(4)+C(5)+C(6)+C(7)+C(8)+C(9)+C(10)+C(11)
03520+    +C(12)+C(13)+C(14)+C(15)+C(16)+C(17)+C(18)+C(19)+C(20)
03530 CI=0.577215665+ALOG(ARG)+SA
03540 CIN=-SA
03550 GO TO 40
03560C*************************************************
03570C*  COMPUTE THE INTEGRALS FOR 1<ARG<INFINITY   *
03580C*************************************************
03590 25 FZ=(ARG**8+38.027264*(ARG**6)+265.187033*(ARG**4)
03600+       +335.67732*(ARG**2)+38.102495)/(ARG*((ARG**8)
03610+       +40.021433*(ARG**6)+322.624911*(ARG**4)
03620+       +570.23628*(ARG**2)+157.105423))
03630 GZ=(ARG**8+42.242855*(ARG**6)+302.757865*(ARG**4)
03640+    +352.018498*(ARG**2)+21.821899)/((ARG**2)*((ARG**8)
03650+    +48.196927*(ARG**6)+482.485984*(ARG**4)+1114.978885
03660+    *(ARG**2)+449.690326))
03670 SI=1.570796327-FZ*COS(ARG)-GZ*SIN(ARG)
03680 CI=FZ*SIN(ARG)-GZ*COS(ARG)
03690 CIN=-CI+0.5772157+ALOG(ARG)
03700 40 RETURN
```

```
03710 END
03720C*************************************************************************
03730       SUBROUTINE FILE(ENC,ENC2,L,POLAR,SIGMA,PSI,GAIN)
03740C*     WRITES DATA ON TAPE50 FOR PLOTTING USING PROCEDURE FILE 4T *
03750C*************************************************************************
03760 N=AINT(10./ENC+80./(ENC+ENC2))
03770 10 FORMAT(I5)
03780 20 FORMAT(2F10.3)
03790 30 FORMAT(F10.3)
03800 40 FORMAT(E9.3)
03810 IF(ENC.GT.0) WRITE(50,10)N
03820 IF(ENC.GT.0) WRITE(50,30)L
03830 IF(ENC.GT.0) WRITE(50,30)POLAR
03840 IF(ENC.GT.0) WRITE(50,40)SIGMA
03850 IF(ENC.LT.0) WRITE(50,20)PSI,GAIN
03860 RETURN
03870 END
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